MHT CET 2023 : 12th May Evening Shift

Mathematics

Question 1

The equation of the line, passing through $(1,2,3)$ and parallel to planes $x-y+2z=5$ and $3x+y+z=6$, is

Options:

A. $\frac{x-1}{-3}=\frac{y-2}{5}=\frac{z-3}{4}$

B. $\frac{x-1}{-3}=\frac{y-2}{-5}=\frac{z-3}{4}$

C. $\frac{x-1}{4}=\frac{y-2}{5}=\frac{z-3}{3}$

D. $\frac{x-1}{5}=\frac{y-2}{7}=\frac{z-3}{1}$

Answer: A

Solution:

Required equation of line is

$$\frac{x-1}{\begin{vmatrix}-1 & 2\\ 1 & 1\end{vmatrix}}=\frac{y-2}{-\begin{vmatrix}1 & 2\\ 3 & 1\end{vmatrix}}=\frac{z-3}{\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}}$$

$$\therefore \quad \frac{x-1}{-3}=\frac{y-2}{5}=\frac{z-3}{4}$$

Question 2

$\lim_{x \to 0} \frac{x \cot 4x}{\sin^2 x \cdot \cot^2(2x)}$ is equal to

Options:

A. 0

B. 1

C. 4

D. $\frac{1}{4}$

Answer: B

Solution:

$$\begin{aligned}\lim_{x \to 0} \frac{x \cot 4x}{\sin^2 x \cot^2(2x)} &= \lim_{x \to 0} \frac{x \tan^2 2x}{\sin^2 x \tan 4x} \\ &= \lim_{x \to 0} \frac{4\left(\frac{\tan 2x}{2x}\right)^2}{4\left(\frac{\sin x}{x}\right)^2\left(\frac{\tan 4x}{4x}\right)} \\ &= 1\end{aligned}$$

Question 3

$\int \frac{1}{\cos^3 x \sqrt{\sin 2x}} dx =$

Options:

A. $\sqrt{2}\left(\sqrt{\tan x} + \frac{1}{5}(\tan x)^{\frac{5}{2}}\right) + \text{c}$, where c is a constant of integration.

B. $\left(\sqrt{\tan x}+\frac{2}{5}(\tan x)^{\frac{5}{2}}\right)+c$, where c is a constant of integration.

C. $\frac{1}{\sqrt{2}}\left(\sqrt{\tan x}+\frac{2}{5}(\tan x)^{\frac{5}{2}}\right)+\mathrm{c}$, where c is a constant of integration.

D. $2\left(\sqrt{\tan x}+\frac{1}{5}(\tan x)^{\frac{5}{2}}\right)+\mathrm{c}$, where c is a constant of integration.

Answer: A

Solution:

$$\begin{aligned}
\text{Let}\quad I &= \int \frac{1}{\cos^3 x\sqrt{\sin 2x}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{\sec^3 x}{\sqrt{\sin x \cos x}} d \\
&= \frac{1}{\sqrt{2}} \int \frac{\sec^4 x}{\sqrt{\tan x}} dx
\end{aligned}$$

Let $\tan x = t \Rightarrow \sec^2 x = dt$

$$\begin{aligned}
\therefore \quad I &= \frac{1}{\sqrt{2}} \int \frac{1+t^2}{\sqrt{t}} dt \\
&= \frac{1}{\sqrt{2}} \int t^{\frac{1}{2}} dt + \frac{1}{\sqrt{2}} \int t^{\frac{3}{2}} dt \\
&= \frac{1}{\sqrt{2}}\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right) + c \\
&= \sqrt{2}\left(\sqrt{\tan x} + \frac{1}{5}(\tan x)^{\frac{5}{2}}\right) + c
\end{aligned}$$

Question 4

The shortest distance (in units) between the lines $\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$ and $\bar{r}=(2\hat{i}-2\hat{j}+3\hat{k})+\lambda(\hat{i}+2\hat{j})$ is

Options:

A. $\frac{8}{3\sqrt{5}}$

B. $\frac{1}{3\sqrt{5}}$

C. $\frac{7}{3\sqrt{5}}$

D. $\frac{2}{3\sqrt{5}}$

Answer: A

Solution:

Given lines are: $\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{0}$

$\therefore$ Required distance

$$=\left|\frac{\begin{vmatrix}3 & 0 & 4\\ 3 & 1 & 2\\ 1 & 2 & 0\end{vmatrix}}{\sqrt{(6-1)^2+(0-2)^2+(0-4)^2}}\right|$$

$$=\left|\frac{3(0-4)+0+4(6-1)}{\sqrt{25+4+16}}\right|$$

$$=\left|\frac{8}{\sqrt{45}}\right|$$

$$=\frac{8}{3\sqrt{5}}$$

Question 5

The maximum value of $z=7x+8y$ subject to the constraints $x+y\leq 20, y\geq 5, x\leq 10, x\geq 0, y\geq 0$ is

Options:

A. 150

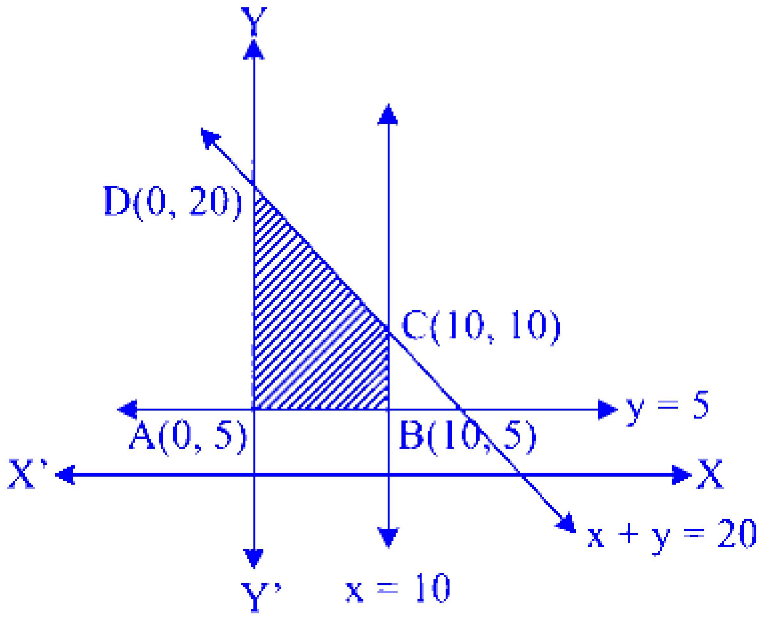
B. 160

C. 110

D. 180

Answer: B

Solution:



Feasible region lies on the origin side of lines $x+y=20, x=10$ and on non-origin side of $y=5$.

$\therefore$ Corner points of the feasible region ant $\mathrm{A}(0,5), \mathrm{B}(10,5), \mathrm{C}(10,10)$ and $\mathrm{D}(0,20)$

z at $\mathrm{A}(0,5)=40$

z at $\mathrm{B}(10,5)=110$

z at $\mathrm{C}(10,10)=150$

z at $\mathrm{D}(0,20)=160$

$\therefore$ Maximum value of z is 160.

Question 6

The value of $\int_0^\pi \left|\sin x-\frac{2x}{\pi}\right| \mathrm{d}x$ is

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: B

Solution:

$$\int_0^{\pi}\left|\sin x-\frac{2x}{\pi}\right|dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\sin x-\frac{2x}{\pi}\right)dx+\int_{\frac{\pi}{2}}^{\pi}\left(\frac{2x}{\pi}-\sin x\right)dx$$

$$=[-\cos x]_0^{\frac{\pi}{2}}-\left[\frac{x^2}{\pi}\right]_0^{\frac{\pi}{2}}+\left[\frac{x^2}{\pi}\right]_{\frac{\pi}{2}}^{\pi}+[\cos x]_{\frac{\pi}{2}}^{\pi}$$

$$=(0+1)-\left(\frac{\pi}{4}-0\right)+\left(\pi-\frac{\pi}{4}\right)+(-1-0)$$

$$=\frac{\pi}{2}$$

Question 7

If $\mathrm{f}(x)=\sin^{-1}\left(\frac{2\log x}{1+(\log x)^2}\right)$, then $\mathrm{f}'(\mathrm{e})$ is

Options:

A. $\frac{2}{\mathrm{e}}$

B. $\frac{1}{2\mathrm{e}}$

C. e

D. $\frac{1}{\mathrm{e}}$

Answer: D

Solution:

$$f(x)=\sin^{-1}\left(\frac{2\log x}{1+(\log x)^2}\right)$$

$$\therefore \quad f'(x)=\frac{1}{\sqrt{1-\left(\frac{2\log x}{1+(\log x)^2}\right)^2}}$$

$$=\frac{1+(\log x)^2}{\sqrt{1+(\log x)^4+2(\log x)^2-4(\log x)^2}}\times\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2\log x}{1+(\log x)^2}\right)$$

$$=\frac{1+(\log x)^2}{\sqrt{1-2(\log x)^2+(\log x)^4}}$$

$$=\frac{1}{1-(\log x)^2}\times\frac{2+2(\log x)^2-4(\log x)^2}{x\left[1+(\log x)^2\right]}$$

$$=\frac{1}{1-(\log x)^2]\times\frac{2}{x}-(2\log x)\left(\frac{2\log x}{x}\right)}\times\frac{2\left[1-(\log x)^2\right]}{x\left[1+(\log x)^2\right]}$$

$$=\frac{2}{x\left[1+(\log x)^2\right]}$$

$$\therefore \quad \mathrm{f}'(\mathrm{e})=\frac{1}{\mathrm{e}}$$

Question 8

If the pair of lines given by $(x\cos\alpha+y\sin\alpha)^2=\left(x^2+y^2\right)\sin^2\alpha$ are perpendicular to each other, then α is

Options:

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

$$(x \cos \alpha + y \sin \alpha)^2 = (x^2 + y^2) \sin^2 \alpha$$

$$\therefore \quad x^2 \cos^2 \alpha + y^2 \sin^2 \alpha + 2xy \sin \alpha \cos \alpha = x^2 \sin^2 \alpha + y^2 \sin^2 \alpha$$

$$\therefore \quad x^2 (\cos^2 \alpha - \sin^2 \alpha) + 2xy \sin \alpha \cos \alpha = 0$$

This represents a pair of straight lines where $\mathrm{a} = \cos^2 \alpha - \sin^2 \alpha, \mathrm{h} = \sin \alpha \cos \alpha$ and $\mathrm{b} = 0$

As lines are perpendicular, we get $a + b = 0$

$\therefore \cos^2 \alpha = \sin^2 \alpha \Rightarrow \alpha = \frac{\pi}{4}$

Question 9

The solution of $\mathrm{e}^{y-x} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y(\sin x + \cos x)}{(1 + y \log y)}$ is

Options:

A. $\frac{\mathrm{e}^y}{y} = \mathrm{e}^x \sin x + \mathrm{c}$, where c is a constant of integration.

B. $\mathrm{e}^y \log y = \mathrm{e}^x \cos x + \mathrm{c}$, where c is a constant of integration.

C. $\mathrm{e}^y \log y = \mathrm{e}^x \sin x + \mathrm{c}$, where c is a constant of integration.

D. $\mathrm{e}^y y = \mathrm{e}^x \sin x + \mathrm{c}$, where c is a constant of integration.

Answer: C

Solution:

$$\mathrm{e}^{y-x} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y(\sin x + \cos x)}{(1 + y \log y)}$$

$$\therefore \quad \mathrm{e}^y \frac{(1 + y \log y)}{y} \mathrm{d}y = \mathrm{e}^x (\sin x + \cos x) \mathrm{d}x$$

$$\therefore \quad \int \mathrm{e}^y \left(\log y + \frac{1}{y}\right) \mathrm{d}y = \int \mathrm{e}^x (\sin x + \cos x) \mathrm{d}x$$

$$\Rightarrow \mathrm{e}^y \log y = \mathrm{e}^x \sin x + \mathrm{c}$$

$$\ldots \left[\because \int \mathrm{e}^x \left(\mathrm{f}(x) + \mathrm{f}'(x)\right) \mathrm{d}x = \mathrm{e}^x \mathrm{f}(x) + \mathrm{c}\right]$$

Question 10

For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ is equal to

Options:

A. $\frac{x \log_e 2x + \log_e 2}{x}$

B. $\frac{x \log_e 2x - \log_e 2}{x}$

C. $x \log_e 2x + \frac{\log_e 2}{x}$

D. $x \log_e 2x - \frac{\log_e 2}{2}$

Answer: B

Solution:

$(2x)^{2y} = 4e^{2x-2y}$

Taking '$\log_e$' on both sides, we get

$2y \log_e(2x) = \log_e 4 + (2x - 2y) \log_e e$
$y \log_e(2x) = \log_e 2 + x - y \quad \ldots\ldots \text{(i)}$

Differentiating w.r.t. x, we get

$[\log_e 2x] \frac{dy}{dx} + \frac{2y}{2x} = 0 + 1 - \frac{dy}{dx}$

$[1 + \log_e 2x] \frac{dy}{dx} = 1 - \frac{y}{x} \quad \ldots. \text{(ii)}$

Now, (i) $\Rightarrow y = \frac{\log_e 2 + x}{1 + \log_e 2x}$

$\therefore \quad$ (ii) $\Rightarrow (1 + \log_e 2x) \frac{dy}{dx} = \frac{x \log_e 2x - \log_e 2}{x(1 + \log_e 2x)}$

$\therefore \quad (1 + \log_e 2x)^2 \frac{dy}{dx} = \frac{x \log_e 2x - \log_e 2}{x}$

Question 11

A poster is to be printed on a rectangular sheet of paper of area 18 m^2. The margins at the top and bottom of 75 cm each and at the sides 50 cm each are to be left. Then the dimensions i.e. height and breadth

of the sheet, so that the space available for printing is maximum, are ________ respectively.

Options:

A. $2\sqrt{3}$ m, $3\sqrt{3}$ m

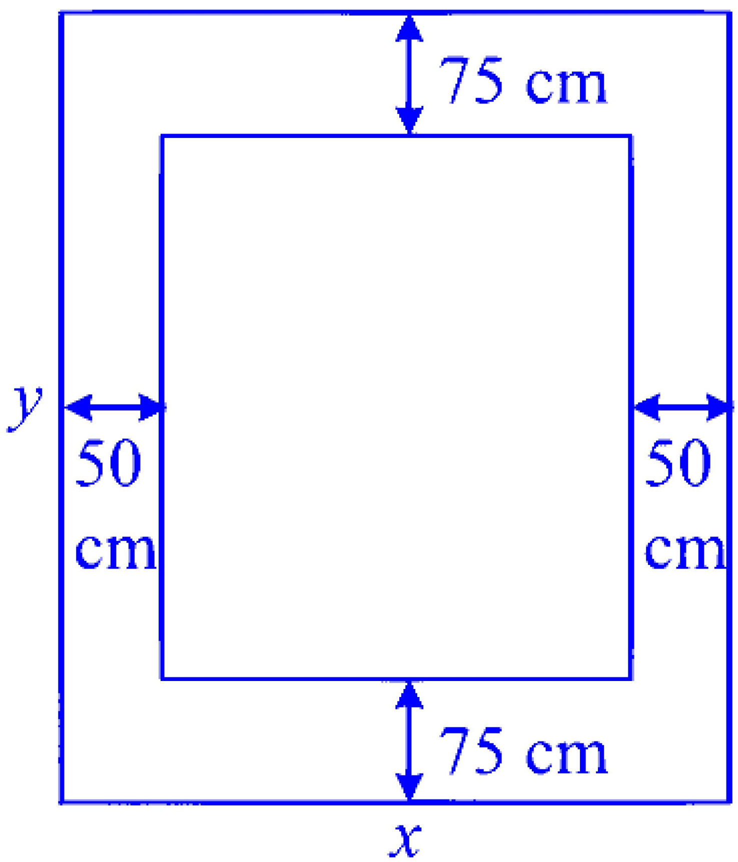
B. $3\sqrt{3}$ m, $2\sqrt{3}$ m

C. 3 m, 6 m

D. 6 m, 3 m

Answer: B

Solution:



Let height and breadth of the sheet be 'y' m and 'x' m respectively.

$\therefore \quad xy = 180000 \text{ cm}^2$

$\therefore \quad y = \frac{180000}{x}$

$\therefore$ The area available for printing is

$$\begin{aligned} A &= (y-150)(x-100) \\ &= \left(\frac{180000}{x} - 150\right)(x-100) \\ &= 180000 - \frac{18000000}{x} - 150x - 15000 \\ &= 165000 - 150x - \frac{18000000}{x} \end{aligned}$$

$\therefore \quad \frac{dA}{dx} = 0 - 150 + \frac{18000000}{x^2}$

$\therefore \quad \frac{dA}{dx} = 0 \Rightarrow x^2 = \frac{18000000}{150} = 120000$

$\Rightarrow x = 200\sqrt{3}$ cm

$\Rightarrow y = \frac{180000}{200\sqrt{3}} = 300\sqrt{3}$ cm

Now, $\frac{d^2A}{dx^2} = \frac{-36000000}{x^3}$

$\therefore$ At $x = 200\sqrt{3}$ cm, $\frac{d^2A}{dx^2} < 0$

$\therefore$ Area is maximum at $x = 200\sqrt{3}$ cm and $y = 300\sqrt{3}$ cm

$\therefore \quad y = 3\sqrt{3}$ m and $x = 2\sqrt{3}$ m

Question 12

The equation of the normal to the curve $3x^2 - y^2 = 8$, which is parallel to the line $x + 3y = 10$, is

Options:

A. $x + 3y + 6 = 0$

B. $x + 3y - 3 = 0$

C. $x + 3y + 8 = 0$

D. $x + 3y - 4 = 0$

Answer: C

Solution:

$3x^2 - y^2 = 8$

Differentiating w.r.t. x, we get

$$6x - 2y\frac{dy}{dx} = 0$$

$$\therefore \quad \frac{dy}{dx} = \frac{3x}{y}$$

$\therefore$ Slope of the tangent to the curve is $\frac{3x}{y}$.

$\therefore$ Slope of the normal is $\frac{-y}{3x}$.

It is parallel to line $x + 3y = 10 \Rightarrow$ slope $= -\frac{1}{3}$

$\therefore$ $\frac{-y}{3x} = \frac{-1}{3} \Rightarrow x = y$

$\therefore$ When $x = y$, equation of the curve becomes $3x^2 - x^2 = 8$

$\therefore$ $x^2 = 4$

$\therefore$ $x = 2, -2 \Rightarrow y = 2, -2$

$\therefore$ $(2, 2)$ and $(-2, -2)$ are the points of contact of the normal and the curve.

$\therefore$ Equations are $(y - 2) = \dfrac{-1}{3}(x - 2)$ or

$(y + 2) = \dfrac{-1}{3}(x + 2)$

i.e., $x + 3y - 8 = 0$ or $x + 3y + 8 = 0$

Question 13

An irregular six faced die is thrown and the probability that, in 5 throws it will give 3 even numbers is twice the probability that it will give 2 even numbers. The number of times, in 6804 sets of 5 throws, you expect to give no even number is

Options:

A. 18

B. 28

C. 27

D. 19

Answer: B

Solution:

Let p be the probability of getting even number.

Let random variable $X \sim B(n, p)$

Given that $P(X=3)=2P(X=2)$

$\therefore \quad {}^5C_3p^3q^2 = 2\,{}^5C_2p^2q^3$

$\therefore \quad p = 2q$

$\therefore \quad p+q=1 \Rightarrow p=\frac{2}{3}$ and $q=\frac{1}{3}$

$\therefore \quad P(X=0) = {}^5C_0p^0q^5 = \frac{1}{3^5}$

$\therefore \quad$ In 1 set of 5 throws, number of times getting no even number is $\frac{1}{3^5}$.

$\therefore \quad$ In 6804 sets of 5 throws, number of times getting no even number is $\frac{1}{3^5} \times 6804 = 28$

Question 14

If the curves $y^2=6x$ and $9x^2+by^2=16$ intersect each other at right angle, then value of 'b' is

Options:

A. $\frac{9}{2}$

B. 6

C. $\frac{7}{2}$

D. 4

Answer: A

Solution:

$y^2 = 6x$ .... (i)

$\Rightarrow 2y\frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = \frac{3}{y}$

Also, $9x^2 + by^2 = 16$

$\Rightarrow 18x + 2by\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-9x}{by}$

As given curves intersect each other at righ angle, their tangents also intersect at right angles.

$\frac{3}{y} \times \frac{-9x}{\text{by}} = -1$

$\Rightarrow \text{by}^2 = 27x$

$\therefore \quad (i) \Rightarrow b(6x) = 27x$

$\Rightarrow b = \frac{9}{2}$

Question 15

The circles $x^2 + y^2 + 2\text{a}x + \text{c} = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other externally, if

Options:

A. $\frac{1}{\text{a}^2} - \frac{1}{\text{b}^2} = \frac{1}{\text{c}}$

B. $\frac{1}{\text{a}^2} + \frac{1}{\text{b}^2} = \frac{1}{\text{c}}$

C. $\frac{1}{\text{a}^2} + \frac{1}{\text{b}^2} = \frac{1}{\text{c}^2}$

D. $\frac{1}{\text{a}^2} - \frac{1}{\text{b}^2} = \frac{1}{\text{c}^2}$

Answer: B

Solution:

$x^2 + y^2 + 2ax + c = 0$

$\Rightarrow x^2 + 2ax + a^2 + y^2 = a^2 - c$

$\Rightarrow (x + a)^2 + y^2 = \left(\sqrt{a^2 - c}\right)^2$

i.e., it is a circle with centre $(-a, 0)$ and radius $\sqrt{\text{a}^2 - \text{c}}$

Similarly,

$$x^2 + y^2 + 2\,\text{by} + \text{c} = 0$$
$$\Rightarrow x^2 + (y + \text{b})^2 = \left(\sqrt{\text{b}^2 - \text{c}}\right)^2$$

i.e., it is a circle with centre $(0, -b)$ and radius $= \sqrt{b^2 - c}$

$\therefore$ If circles touch externally, then we get

Sum of radii $=$ Distance between centres

$$\Rightarrow \sqrt{a^2 - c} + \sqrt{b^2 - c} = \sqrt{a^2 + b^2}$$
$$\Rightarrow a^2 - c + b^2 - c + 2\sqrt{a^2 - c}\sqrt{b^2 - c} = a^2 + b^2$$
$$\Rightarrow (a^2 - c)(b^2 - c) = c^2$$
$$\Rightarrow a^2b^2 - cb^2 - ca^2 + c^2 = c^2$$
$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$$

Question 16

Given $\text{f}(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , \text{if } x < 0 \\ \text{a} & , \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16-\sqrt{x}-4}}, & \text{if } x > 0 \end{cases}$

If $\text{f}(x)$ **is continuous at** $x = 0$**, then value of a is**

Options:

A. -8

B. 2

C. -2

D. 8

Answer: D

Solution:

As $\mathrm{f}(x)$ is continuous at $x=0$, we get

$$\lim_{x \to 0^-} \mathrm{f}(x) = \mathrm{a} = \lim_{x \to 0^+} \mathrm{f}(x)$$

$\therefore \quad \lim_{x \to 0^-} \mathrm{f}(x) = \mathrm{a}$

$\therefore \quad \lim_{x \to 0} \frac{1-\cos 4x}{x^2} = \mathrm{a}$

$\therefore \quad \lim_{x \to 0} 4 \frac{2\sin^2 2x}{(2x)^2} = \mathrm{a}$

$\therefore \quad 8 = \mathrm{a}$

Question 17

A, B, C, D are four points in a plane with position vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ respectively such that $(\bar{a}-\bar{d}) \cdot (\bar{b}-\bar{c}) = (\bar{b}-\bar{d}) \cdot (\bar{c}-\bar{a}) = 0$. The point D, then is the ____________ of $\triangle \mathrm{ABC}$

Options:

A. centroid

B. circumcentre

C. incentre

D. orthocentre

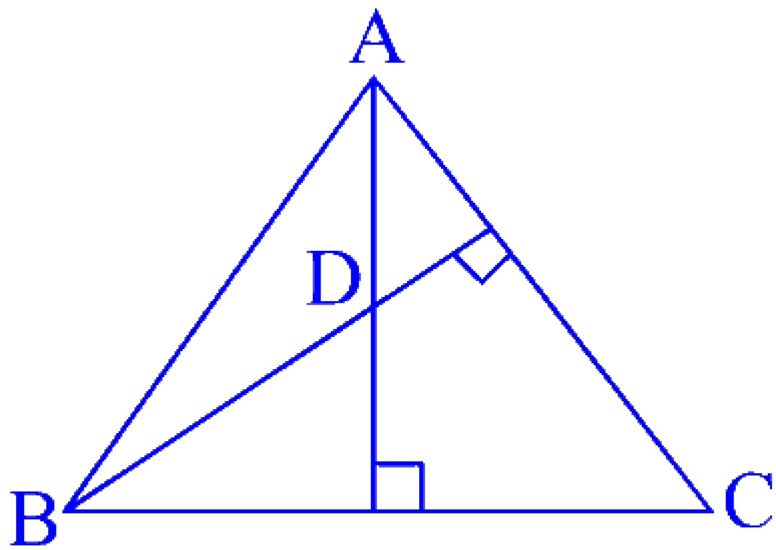
Answer: D

Solution:

$(\bar{a}-\bar{d}) \cdot (\bar{b}-\bar{c}) = (\bar{b}-\bar{d})(\bar{c}-\bar{a}) = 0$

$\overline{AD} \cdot \overline{BC} = \overline{BD} \cdot \overline{CA} = 0$

$\Rightarrow \overline{AD} \perp \overline{BC}$ and $\overline{BD} \perp \overline{CA}$



$\Rightarrow D$ is the orthocentre of $\triangle ABC$.

Question 18

Two adjacent of sides parallelogram ABCD are given by $\overline{\mathrm{AB}} = 2\hat{\mathrm{i}} + 10\hat{\mathrm{j}} + 11\hat{\mathrm{k}}$ and $\overline{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by angle α in plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

Options:

A. $\frac{8}{9}$

B. $\frac{1}{9}$

C. $\frac{\sqrt{17}}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: C

Solution:

Let θ be the angle between $\overline{\mathrm{AB}}$ and $\overline{\mathrm{AD}}$

$$\therefore \quad \cos\theta = \frac{\overline{AB}\cdot\overline{AD}}{|\overline{AB}|\overline{AD}\,|}$$

$$= \frac{(2\hat{i}+10\hat{j}+11\hat{k})\cdot(-\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{4+100+121}\sqrt{1+4+4}}$$

$$= \frac{-2+20+22}{\sqrt{225}\sqrt{9}}$$

$$= \frac{40}{45}$$

$$= \frac{8}{9}$$

$$\therefore \quad \sin\theta = \sqrt{1-\left(\frac{8}{9}\right)^2} = \frac{\sqrt{17}}{9}$$

α is the angle of rotation of AD The angle between side AB and AD

$= \alpha + \theta$

$= 90^\circ$ ...[Given]

$$\therefore \quad \cos(\alpha+\theta) = \cos\left(90^\circ\right)$$

$$\therefore \quad \cos\alpha\cos\theta - \sin\alpha\sin\theta = 0$$

$$\therefore \quad 8\cos\alpha = \sqrt{17}\sin\alpha$$

$$\therefore \quad 64\cos^2\alpha = 17\left(1-\cos^2\alpha\right)$$

$$\therefore \quad 81\cos^2\alpha = 17$$

$$\therefore \quad \cos\alpha = \tfrac{\sqrt{17}}{9}$$

Question 19

The value of x, for which $\sin\left(\cot^{-1}(x)\right) = \cos\left(\tan^{-1}(1+x)\right)$, is

Options:

A. 0

B. 1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

Answer: C

Solution:

Note that $\cot^{-1} x = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$ and

$$\tan^{-1}(1+x) = \cos^{-1}\left(\frac{1}{\sqrt{1+(1+x)^2}}\right)$$

$$\therefore \quad \sin\left(\cot^{-1}(x)\right) = \cos\left(\tan^{-1}(1+x)\right)$$

$$\Rightarrow \sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+(1+x)^2}}\right)\right)$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+(1+x)^2}}$$

$$\Rightarrow 1+(1+x)^2 = 1+x^2$$

$$\Rightarrow x = \frac{-1}{2}$$

Question 20

The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ is

Options:

A. $\frac{8\hat{i}-3\hat{j}+3\hat{k}}{\sqrt{82}}$

B. $\frac{-8\hat{i}-3\hat{j}+3\hat{k}}{\sqrt{82}}$

C. $\frac{-8\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{82}}$

D. $-\frac{8\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{82}}$

Answer: C

Solution:

Consider option (C)

$(3\hat{i}+2\hat{j}+6\hat{k})\cdot\left(\frac{-8\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{82}}\right)=0$

This is valid for only option (C)

$\therefore$ Option (c) is correct.

Question 21

A ladder, 5 meters long, rests against a vertical wall. If its top slides downwards at the rate of $10\ \mathrm{cm/s}$, then the angle between the ladder and the floor is decreasing at the rate of ____________ radians/second when it's lower end is $4\ \mathrm{m}$ away from the wall.

Options:

A. -0.1

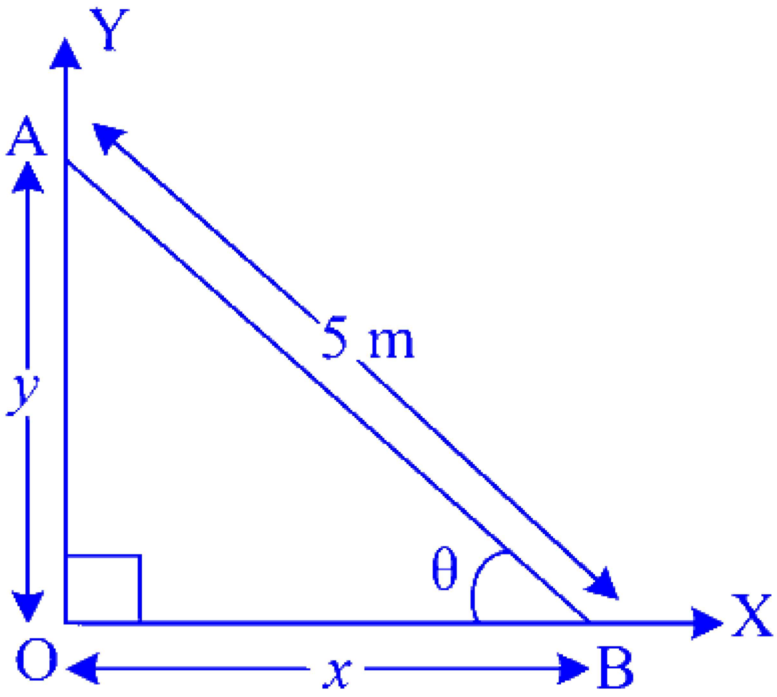
B. -0.025

C. 0.1

D. 0.025

Answer: D

Solution:



According to the figure, $x^2+y^2=25$ .... (i)

Note that $\cos\theta=\frac{\mathrm{OB}}{\mathrm{AB}}=\frac{x}{5}$

$\therefore \quad x=5\cos\theta$

$\therefore \quad \text{(i)} \Rightarrow 25\cos^2\theta+y^2=25$

Differentiating w.r.t. 't', we get

$$-50\cos\theta\sin\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}+2y\frac{\mathrm{d}y}{\mathrm{dt}}=0$$

$$25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=y\frac{\mathrm{d}y}{\mathrm{dt}}$$

$$\therefore \quad 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=y(-0.1)$$

$$\ldots\left[\because \frac{\mathrm{d}y}{\mathrm{d}x}=-10\ \mathrm{cm/s}=-0.1\ \mathrm{m/s}\right]$$

$\therefore \quad 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=-(0.1)y$ .... (ii)

at $x=4$, $\cos\theta=\frac{4}{5}$, $\sin\theta=\frac{3}{5}$ and $y=3$

$\therefore \quad \text{(ii)} \Rightarrow 25\times\frac{3}{5}\times\frac{4}{5}\times\frac{\mathrm{d}\theta}{\mathrm{dt}}=-0.3$

$\Rightarrow \frac{\mathrm{d}\theta}{\mathrm{dt}}=-0.025$

i.e., the angle is decreasing at the rate of $0.025\ \mathrm{rad/s}$

Question 22

If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of them are collinear, $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+2\vec{b}$ is

Options:

A. $6\overline{\mathrm{c}}$

B. $-6\overline{\mathrm{c}}$

C. $\bar{c}$

D. $2\overline{\mathrm{c}}$

Answer: B

Solution:

$\overline{\mathrm{a}}+2\overline{\mathrm{b}}$ is collinear with $\overline{\mathrm{c}}$

$\therefore$ $\overline{\mathrm{a}}+2\overline{\mathrm{b}}=\mathrm{nc}$ .... (i)

Similarly $\overline{\mathrm{b}}+3\overline{\mathrm{c}}=\mathrm{ma}$ .... (ii)

m and n are non-zero scalars.

$\therefore$ (i) $\Rightarrow \overline{\mathrm{a}}+2\overline{\mathrm{b}}+6\overline{\mathrm{c}}=(\mathrm{n}+6)\overline{\mathrm{c}}$

(ii) $\Rightarrow \overline{\mathrm{a}}+2\overline{\mathrm{b}}+6\overline{\mathrm{c}}=(2\mathrm{~m}+1)\overline{\mathrm{a}}$

$\Rightarrow \mathrm{n}+6=0$ and $2\mathrm{~m}+1=0$

$\Rightarrow \mathrm{n}=-6$ and $\mathrm{m}=\frac{-1}{2}$

$\therefore$ (i) $\Rightarrow \vec{a}+2\vec{b}=-6\overline{\mathrm{c}}$

Question 23

If $|z-2+i| \leq 2$, then the difference between the greatest and least value of $|z|$ is _________, $(\mathrm{i}=\sqrt{-1})$

Options:

A. $2\sqrt{5}+4$

B. $2\sqrt{5}$

C. 4

D. 8

Answer: C

Solution:

Note that $|z_1 - z_2| \geq ||z_1| - |z_2||$

$$\begin{aligned} \therefore \quad & |z - 2 + i| \leq 2 \\ & \Rightarrow ||z| - |2 - i|| \leq 2 \\ & \Rightarrow -2 \leq |z| - |2 - i| \leq 2 \end{aligned}$$

$$\begin{aligned} & \Rightarrow -2 \leq |z| - \sqrt{4+1} \leq 2 \\ & \Rightarrow -2 \leq |z| - \sqrt{5} \leq 2 \\ & \Rightarrow \sqrt{5} - 2 \leq |z| \leq 2 + \sqrt{5} \end{aligned}$$

$\Rightarrow$ Largest value of $|z|$ is '$2 + \sqrt{5}$' and the least value is '$\sqrt{5} - 2$'

$\therefore \quad$ Required difference $= 2 + \sqrt{5} - (\sqrt{5} - 2) = 4$

Question 24

A box contains 100 tickets numbered 1 to 100 . A ticket is drawn at random from the box. Then the probability, that number on the ticket is a perfect square, is

Options:

A. $\frac{1}{10}$

B. $\frac{3}{10}$

C. $\frac{7}{100}$

D. $\frac{9}{100}$

Answer: A

Solution:

Let X : Event that number on the ticket is perfect square.

$\therefore \quad X = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$\therefore \quad n(X) = 10$

Also, $n(S) = 100$

$\therefore \quad$ Required probability $= \frac{n(X)}{n(S)} = \frac{10}{100} = \frac{1}{10}$

Question 25

If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)\left(\sqrt{1-x^2}\right)^m + c$ for a suitable chosen integer m and a function $A(x)$, where c is a constant of integration, then $(A(x))^m$ equals

Options:

A. $\frac{1}{9x^4}$

B. $\frac{-1}{3x^3}$

C. $\frac{-1}{27x^9}$

D. $\frac{1}{27x^6}$

Answer: C

Solution:

$$\text{Let} \quad I = \int \frac{\sqrt{1-x^2}}{x^4} dx$$

$$= \int \frac{x\sqrt{\frac{1}{x^2} - 1}}{x^4} dx$$

$$= \int \frac{\sqrt{\frac{1}{x^2} - 1}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = t$

$$\therefore \quad \frac{-2}{x^3}\,dx = dt \Rightarrow \frac{1}{x^3}\,dx = \frac{-dt}{2}$$

$$\therefore \quad I = -\frac{1}{2}\int \sqrt{t}\,dt$$

$$= \frac{-1}{2} \times \frac{(t)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{-1}{3} \times \left(\frac{1}{x^2} - 1\right)^{\frac{3}{2}} + c$$

$$= \frac{-1}{3} \times \frac{\left(1 - x^2\right)^{\frac{3}{2}}}{\left(x^2\right)^{\frac{3}{2}}} + c$$

$$= \frac{-1}{3} \times \frac{\left(\sqrt{1 - x^2}\right)^3}{x^3}$$

Comparing with $A(x)\left(\sqrt{1-x^2}\right)^m + c$, we get $A(x) = \frac{-1}{3x^3}$ and $m = 3$

$$\therefore \quad (A(x))^m = \left(\frac{-1}{3x^3}\right)^3 = \frac{-1}{27x^9}$$

Question 26

If k_i are possible values of k for which lines $kx + 2y + 2 = 0, 2x + ky + 3 = 0$ and $3x + 3y + k = 0$ are concurrent, then $\sum k_i$ has the value

Options:

A. 0

B. -2

C. 2

D. 5

Answer: A

Solution:

If three lines $a_1x + b_1y + c_1 = 0, a_2 + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} k & 2 & 2 \\ 2 & k & 3 \\ 3 & 3 & k \end{vmatrix} = 0$$

$\Rightarrow k\left(k^2 - 9\right) - 2(2k - 9) + 2(6 - 3k) = 0$

$\Rightarrow k^3 - 9k - 4k + 18 + 12 - 6k = 0$

$\Rightarrow k^3 - 19k + 30 = 0$

$\Rightarrow (k - 2)\left(k^2 + 2k - 15\right) = 0$

$\Rightarrow (k - 2)(k + 5)(k - 3) = 0$

$\Rightarrow k_1 = 2, k_2 = -5$ and $k_3 = 3$

$\Rightarrow \sum k_i = 0$

Question 27

A tank with a rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 4 meter and volume is 36 cubic meters. If building of the tank costs ₹ 100 per square meter for the base and ₹ 50 per square meter for the sides, then the cost of least expensive tank is

Options:

A. ₹ 3000

B. ₹ 3300

C. ₹ 2400

D. ₹ 3500

Answer: B

Solution:

Let length and breadth of the tank be 'x' m and 'y' m respectively.

Height of the tank is 4 m.

Height of the tank

Volume $= 36 \text{ m}^3$

$\therefore \quad 4xy = 36$

$\therefore \quad xy = 9 \quad \ldots \text{(i)}$

$\therefore \quad y = \frac{9}{x} \quad \ldots \text{(ii)}$

$\therefore$ Total area of the tank including sides and base $= xy + 2(4x) + 2(4y)$

$\therefore \mathrm{f}(x) = 9 + 8x + 8\left(\frac{9}{x}\right)$ ...[From (i) and (ii)]

$= 9 + 8x + \frac{72}{x}$

$\therefore \quad f'(x) = 8 - \frac{72}{x^2}$

$\therefore \quad \mathrm{f}'(x) = 0 \Rightarrow x = 3$

$\Rightarrow y = 3$

$\therefore$ Required cost $= 100 \times (3 \times 3) + 50 \times (2 \times 4 \times 3 + 2 \times 4 \times 3)$

$= 900 + 2400$

$= 3300$

Question 28

The length (in units) of the projection of the line segment, joining the points $(5, -1, 4)$ and $(4, -1, 3)$, on the plane $x + y + z = 7$ is

Options:

A. $\frac{2}{\sqrt{3}}$

B. $\frac{2}{3}$

C. $\frac{\sqrt{2}}{3}$

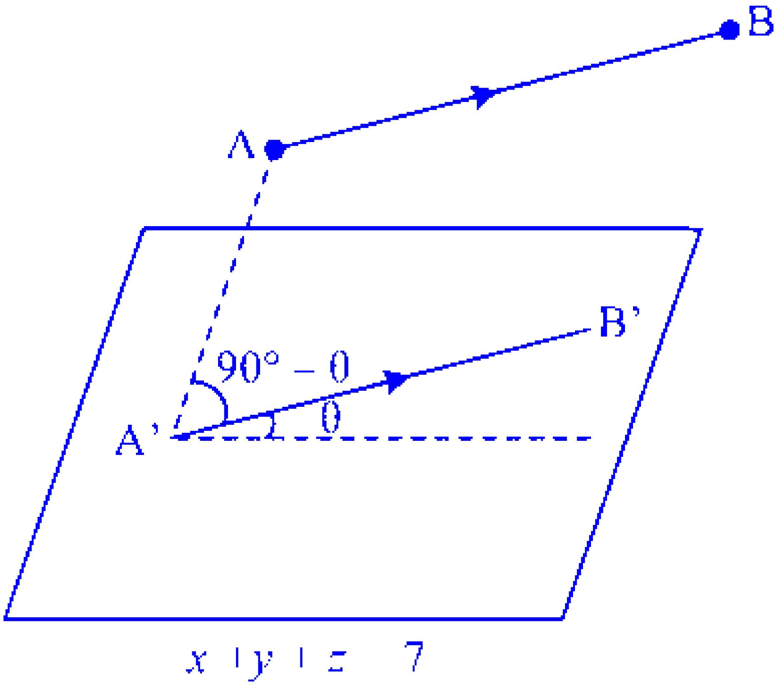
D. $\sqrt{\frac{2}{3}}$

Answer: D

Solution:

Let $A=(5,-1,4), B=(4,-1,3)$

$\overline{\mathrm{AB}}=-\hat{\mathrm{i}}-\hat{\mathrm{k}} \Rightarrow|\overline{\mathrm{AB}}=\sqrt{2}$



Projection of $\overline{\mathrm{AB}}$ in the plane $x+y+z=7$ is $|\overline{\mathrm{AB}}| \cos \theta=\left|\overline{\mathrm{A}'\mathrm{B}'}\right| \cos \theta$

Direction ratios of normal to the given plane is $1,1,1$.

$$\cos \left(90^{\circ}-\theta\right)=\left|\frac{1(-1)+1(0)+1(-1)}{\sqrt{1^2+1^2+1^2} \sqrt{1^2+0^2+1^2}}\right|$$

$$\Rightarrow \sin \theta=\frac{2}{\sqrt{6}} \Rightarrow \cos \theta=\sqrt{1-\frac{4}{6}}=\sqrt{\frac{1}{3}}$$

Required projection $=|\overline{\mathrm{AB}}| \cos \theta$

$$=\sqrt{2} \times \frac{1}{\sqrt{3}}=\sqrt{\frac{2}{3}}$$

Question 29

If the volume of tetrahedron, whose vertices are $\mathrm{A}(1,2,3), \mathrm{B}(-3,-1,1), \mathrm{C}(2,1,3)$ and $D(-1,2,x)$ is $\frac{11}{6}$ cubic units, then the value of x is

Options:

A. 3

B. -2

C. 4

D. -1

Answer: C

Solution:

$$\begin{aligned} &\overline{\mathrm{AB}}=-4\hat{\mathrm{i}}-3\hat{\mathrm{j}}-2\hat{\mathrm{k}} \\ &\overline{\mathrm{AC}}=\hat{\mathrm{i}}-\hat{\mathrm{j}} \\ &\overline{\mathrm{AD}}=-2\hat{\mathrm{i}}+(x-3)\hat{\mathrm{k}} \\ &\text{Volume }=\frac{1}{6}\begin{vmatrix}-4 & -3 & -2 \\ 1 & -1 & 0 \\ -2 & 0 & x-3\end{vmatrix} \\ &11=4(x-3)+3(x-3)-2(-2) \\ \therefore\quad &11=7x-17 \\ \therefore\quad &x=4 \end{aligned}$$

Question 30

Let

Statement 1 : If a quadrilateral is a square, then all of its sides are equal.

Statement 2: All the sides of a quadrilateral are equal, then it is a square.

Options:

A. Statement 2 is contrapositive of statement 1.

B. Statement 2 is negation of statement 1.

C. Statement 2 is inverse of statement 1.

D. Statement 2 is the converse of statement 1.

Answer: D

Solution:

Let p : A quadrilateral is a square

q : All sides of quadrilateral are equal.

$\therefore$ Statement 1 is $p \rightarrow q$,

Statement 2 is $q \rightarrow p$

$\therefore$ Statement 2 is the converse of statement 1.

Question 31

The number of words that can be formed by using the letters of the word CALCULATE such that each word starts and ends with a consonant, are

Options:

A. $5 \times 7!$

B. $\frac{9!}{8}$

C. $\frac{5 \times 7!}{2}$

D. $20 \times 7!$

Answer: C

Solution:

Word CALCULATE has 9 letters. Out of which 'C' repeats 2 times, 'A' repeats 2 times, 'L' repeats 2 times, 'E', 'U' and 'T' repeats once.

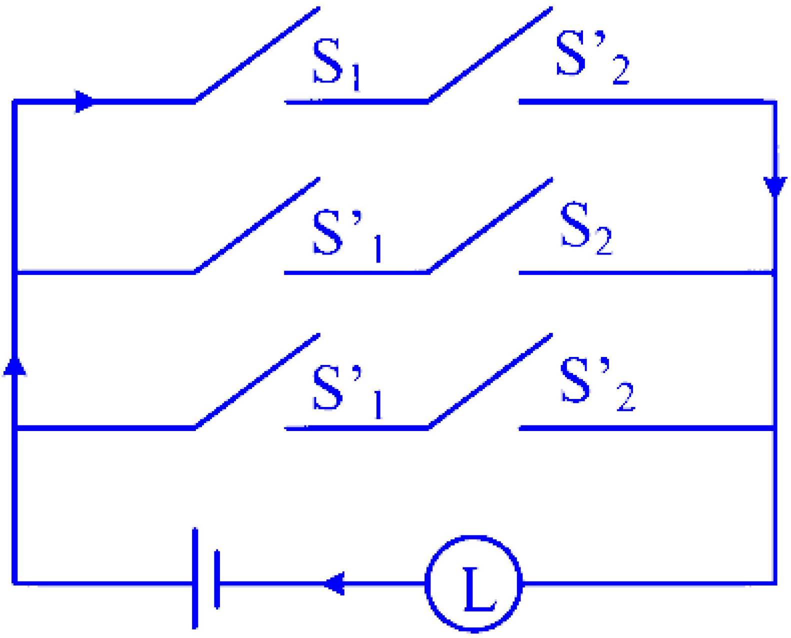
$\therefore$ There are 5 consonants and 4 vowels. Two consonants out of 5 can take start and end position of the word in $^5P_2$ ways.

And remaining 7 letters can take remaining 7 positions in 7 ! ways. Also, 'C', 'A' and 'L' repeats twice each.

$\therefore$ The required number of words that can be formed $= \frac{5p_2 \times 7!}{2! \times 2! \times 2!} = \frac{5 \times 4 \times 3! \times 7!}{3! \times 2 \times 2 \times 2} = \frac{5 \times 7!}{2}$

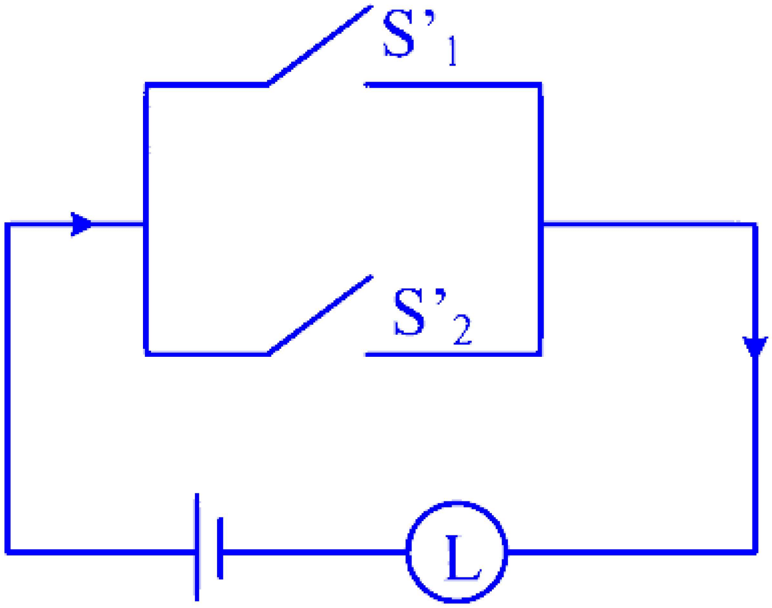
Question 32

The given following circuit is equivalent to

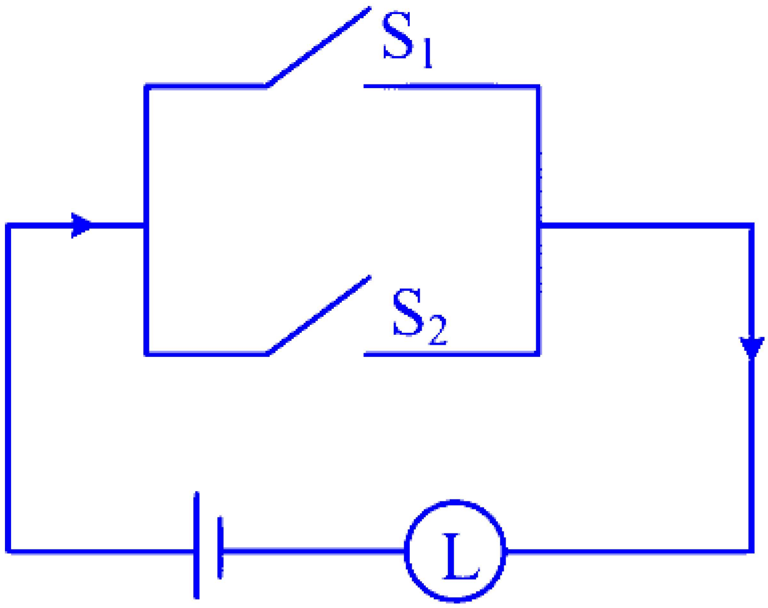


Options:

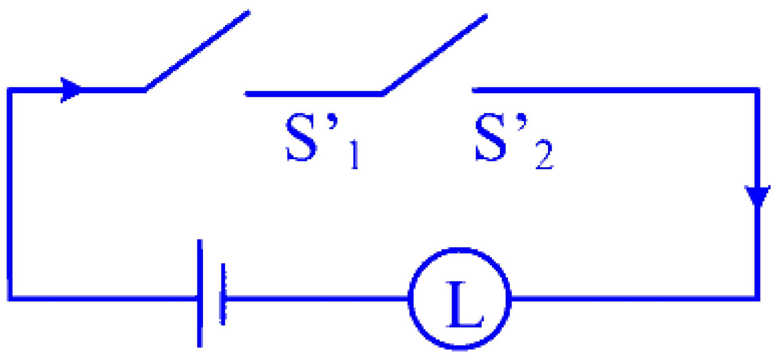
A.



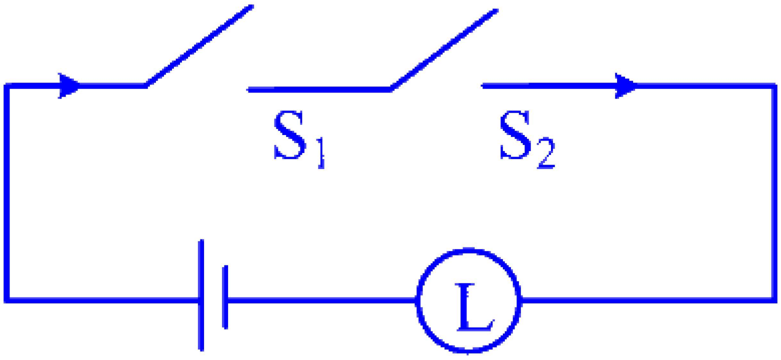
B.



C.



D.



Answer: A

Solution:

Let p : The switch S_1

q : The switch S_2

The symbolic form is

$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$\equiv (p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)]$ ...[Distributive Law]

$\equiv (p \wedge \sim q) \vee [\sim p \wedge t]$ ...[Complement Law]

$\equiv (p \wedge \sim q) \vee \sim p$...[Identity Law]
$\equiv \sim p \vee (p \wedge \sim q)$... [Commutative Law]
$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$...[Distributive Law]

$\equiv t \wedge (\sim p \vee \sim q)$ ...[Complement Law]

$\equiv \sim p \vee \sim q$ ...[Identity Law]

$\therefore$ Option (A) is correct.

Question 33

Water flows from the base of rectangular tank, of depth 16 meters. The rate of flow of the water is proportional to the square root of depth at any time t. If depth is $4 \mathrm{~m}$ when $\mathrm{t}=2$ hours, then after 3.5 hours the depth (in meters) is

Options:

A. 0

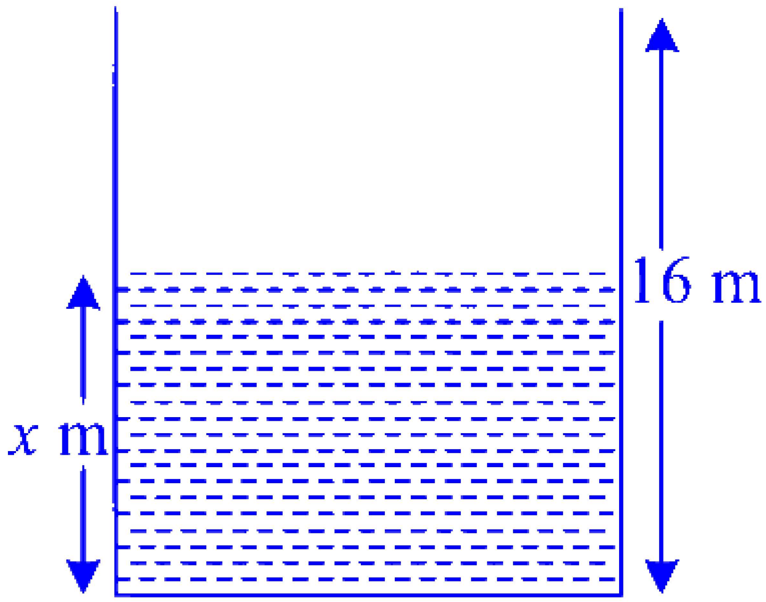
B. 0.25

C. 0.5

D. 3

Answer: B

Solution:



Given that $\frac{dx}{dt} \propto \sqrt{x}$

$\therefore \quad \frac{dx}{dt} = a\sqrt{x}$, for real number a

$\therefore \quad \int \frac{dx}{\sqrt{x}} = \int a dt$

$\therefore \quad 2\sqrt{x} = at + c$ .... (i)

When $t = 0, x = 16$

$\therefore \quad$ (i) $\Rightarrow c = 8$

$\therefore \quad$ (i) becomes $2\sqrt{x} = at + 8$ .... (ii)

When $t = 2, x = 4$

$\therefore \quad$ (ii) $\Rightarrow a = -2$

$\therefore \quad$ (ii) becomes $2\sqrt{x} = -2t + 8$ .... (iii)

$\therefore \quad$ when $t = 3.5$

(iii) $\Rightarrow x = 0.25$ m

Question 34

The area (in sq. units) of the smaller part of the circle $x^2 + y^2 = \mathrm{a}^2$ cut off by the line $x = \frac{\mathrm{a}}{\sqrt{2}}$ is

Options:

A. $\frac{\mathrm{a}^2}{4}\left|\frac{\pi}{2} - 1\right|$

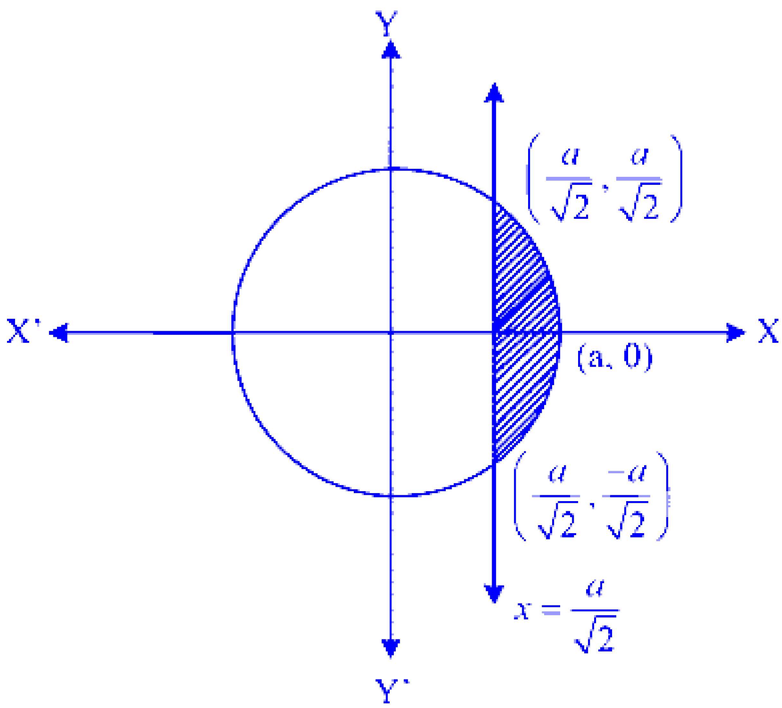
B. $a^2\left|\frac{\pi}{4} - 1\right|$

C. $\frac{\mathrm{a}^2}{2}\left|\frac{\pi}{2} - 1\right|$

D. $\frac{\mathrm{a}^2}{4}\left|\frac{\pi}{4} - 1\right|$

Answer: C

Solution:



Substitute $x = \frac{\mathrm{a}}{\sqrt{2}}$ in $x^2 + y^2 = \mathrm{a}^2$, we get $\frac{\mathrm{a}^2}{2} + y^2 = \mathrm{a}^2 \Rightarrow y = \pm\frac{\mathrm{a}}{\sqrt{2}}$

$\therefore$ Required area

$$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}dx$$

$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{\frac{a}{\sqrt{2}}}^{2}$$

$$= 2\left\{\left[0 + \frac{a^2}{2} \times \frac{\pi}{2}\right] - \left[\frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} + \frac{a^2}{2} \times \frac{\pi}{4}\right]\right\}$$

$$= 2\left[\frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}\right]$$

$$= \frac{a^2}{2}\left|\pi - 1 - \frac{\pi}{2}\right|$$

$$= \frac{a^2}{2}\left|\frac{\pi}{2} - 1\right|$$

Question 35

$\cos^2 48^\circ - \sin^2 12^\circ =$ __________, **if** $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Options:

A. $\frac{-\sqrt{5}+1}{8}$

B. $\frac{\sqrt{5}-1}{8}$

C. $\frac{\sqrt{5}+1}{8}$

D. $\frac{-\sqrt{5}-1}{8}$

Answer: C

Solution:

Let $A=\cos^2 48^\circ-\sin^2 12^\circ$
$=\cos^2(30^\circ+18^\circ)-\sin^2(30^\circ-18^\circ)$
$=[\cos 30^\circ\cos 18^\circ-\sin 30^\circ\sin 18^\circ]^2$
$-[\sin 30^\circ\cos 18^\circ-\sin 18^\circ\cos 30^\circ]^2$

$$=\left[\frac{\sqrt{3}\cos 18^\circ-\sin 18^\circ}{2}\right]^2-\left[\frac{\cos 18^\circ-\sqrt{3}\sin 18^\circ}{2}\right]^2$$

$$=\frac{3\cos^2 18^\circ+\sin^2 18-2\sqrt{3}\sin 18^\circ\cos 18^\circ}{4}$$

$$-\frac{\cos^2 18^\circ+3\sin^2 18-2\sqrt{3}\sin 18^\circ\cos 18^\circ}{4}$$

$$=\frac{\cos^2 18^\circ-\sin^2 18^\circ}{2}$$

Note that $\sin 18^\circ=\frac{\sqrt{5}-1}{4}\Rightarrow\cos^2 18^\circ=\frac{5+\sqrt{5}}{8}$

and $\sin^2 18^\circ=\frac{3-\sqrt{5}}{8}$

$$\therefore\quad A=\frac{\frac{5+\sqrt{5}-3+\sqrt{5}}{8}}{2}=\frac{1+\sqrt{5}}{8}$$

Question 36

The discrete random variable X can take all possible integer values from 1 to k, each with a probability $\frac{1}{k}$, then its variance is

Options:

A. $\frac{k^2-1}{12}$

B. $\frac{k^2-1}{6}$

C. $\frac{k^2+1}{12}$

D. $\frac{k^2+1}{6}$

Answer: A

Solution:

x_i	1	2	3	---	k
p_i	$\frac{1}{k}$	$\frac{1}{k}$	$\frac{1}{k}$	---	$\frac{1}{k}$

$$E(X)=\sum_{i=1}^{k} x_i p_i=\frac{1}{k}+\frac{2}{k}+\ldots+\frac{k}{k}=\frac{k+1}{2}$$

$$E\left(X^2\right)=\sum_{i=1}^{k} x_i^2 p_i=\frac{1^2+2^2+3^2+\ldots+k^2}{k}$$

$$=\frac{(\mathrm{k}+1)(2\mathrm{k}+1)}{6}$$

$$\text{Variance } =E\left(X^2\right)-[E(X)]^2$$

$$=\frac{2k^2+3k+1}{6}-\frac{k^2+2k+1}{4}$$

$$=\frac{4k^2+6k+2-3k^2-6k-3}{12}$$

$$=\frac{k^2-1}{12}$$

Question 37

If $\tan y=\frac{x \sin \alpha}{1-x \cos \alpha}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{m}}{x^2+2\mathrm{n}x+1}$, then $\mathrm{m}^2+\mathrm{n}^2$ is

Options:

A. 2

B. 3

C. 1

D. 4

Answer: C

Solution:

$\tan y=\frac{x\sin\alpha}{1-x\cos\alpha}$

$\therefore \quad y=\tan^{-1}\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$

$\therefore \quad \frac{dy}{dx}=\frac{1}{1+\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)^2}\frac{d}{dx}\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$

$=\frac{1}{\frac{1-2x\cos\alpha+x^2\cos^2\alpha+x^2\sin^2\alpha}{(1-x\cos\alpha)^2}}$

$\times\frac{(1-x\cos\alpha)\sin\alpha+(x\sin\alpha)\cos\alpha}{(1-x\cos\alpha)^2}$

$=\frac{\sin\alpha-x\sin\alpha\cos\alpha+x\sin\alpha\cos\alpha}{1+2(-\cos\alpha)x+x^2}$

$=\frac{\sin\alpha}{x^2+2(-\cos\alpha)+1}$

$=\frac{m}{x^2+2nx+1}$ ... [Given]

$\Rightarrow$ n $=-\cos\alpha$ and m $=\sin\alpha$

$\Rightarrow m^2+n^2=1$

Question 38

The lengths of sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Then the length of the sides of the triangle (in units) are

Options:

A. $3,4,5$

B. $4,5,6$

C. $5,6,7$

D. $2,3,4$

Answer: B

Solution:

Let $a, a+1, a+2$ be the sides of the triangle and A, B, C be the angles opposite to them respectively.

According to the given condition, C $=2$ A

$$\therefore \quad \sin C = \sin 2A$$
$$\sin C = 2 \sin A \cos A \quad .... \text{(i)}$$

Note that $\frac{\sin A}{a} = \frac{\sin C}{a+2} = k$

$\Rightarrow \sin \mathrm{A} = \mathrm{ka}$ and $\sin \mathrm{C} = \mathrm{k}(\mathrm{a}+2)$

$$\text{Also, } \cos A = \frac{(a+1)^2 + (a+2)^2 - a^2}{2(a+1)(a+2)}$$
$$= \frac{a^2 + 2a + 1 + a^2 + 4a + 4 - a^2}{2\left(a^2 + 3a + 2\right)}$$
$$= \frac{a^2 + 6a + 5}{2\left(a^2 + 3a + 2\right)}$$

$\therefore \quad \text{(i)} \Rightarrow k(a+2) = 2 \times ka \times \frac{a^2+6a+5}{2(a^2+3a+2)}$

$\therefore \quad a + 2 = \frac{a\left(a^2+6a+5\right)}{\left(a^2+3a+2\right)}$

$\therefore \quad (a+2)\left(a^2+3a+2\right) = a^3 + 6a^2 + 5a$

$$\therefore \quad a^3 + 5a^2 + 8a + 4 = a^3 + 6a^2 + 5a$$
$$\therefore \quad a^2 - 3a - 4 = 0$$
$$\therefore \quad (a-4)(a+1) = 0$$
$$\Rightarrow a = 4 \text{ or } -1$$

But $\mathrm{a} = -1$ is not possible.

$\therefore$ 4, 5, 6 are the lengths of the sides of the triangle.

Question 39

For 20 observations of variable x, if $\sum(x_i - 2) = 20$ and $\sum(x_i - 2)^2 = 100$, then the standard deviation of variable x is

Options:

A. 2

B. 3

C. 4

D. 9

Answer: A

Solution:

Note that standard derivation is independent of change of origin.

$\therefore \quad$ S.D. of $x_i =$ S.D. of $(x_i - 2)$

$\therefore$ S.D. of $x_i =$ S.D. of $(x_i - 2)$

$$\therefore \quad \text{S.D. of } (x_i - 2) = \sqrt{\frac{1}{n}\sum_{i=1}^{20}(x_i-2)^2 - \left[\frac{\sum(x_i-2)}{n}\right]^2}$$
$$= \sqrt{\frac{100}{20} - (1)^2}$$
$$= 2$$

$\Rightarrow$ Required S.D $= 2$

Question 40

Equation of plane containing the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

Options:

A. $x + 2y + z = 0$

B. $x + 2y - z = 0$

C. $x - 2y + z = 0$

D. $x - 2y - z = 0$

Answer: C

Solution:

Equation of the plane containing $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

$$\begin{vmatrix} x & y & z \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 0$$
$$\Rightarrow 8x - y - 10z = 0$$

Now required plane is perpendicular to this plane.

Consider option (C)

$(8)(1)+(-1)(-2)+(-10)(1)=0$

$\therefore$ Option (C) is correct.

Question 41

If $(2+\sin x)\frac{\mathrm{d}y}{\mathrm{d}x}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is

Options:

A. $\frac{-2}{3}$

B. $\frac{-1}{3}$

C. $\frac{4}{3}$

D. $\frac{1}{3}$

Answer: D

Solution:

$$(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$$
$$\therefore \quad \frac{1}{y+1}dy=\frac{-\cos x}{2+\sin x}dx$$

$\therefore$ Integrating both sides, we get

$$\log(y+1)=-\log(2+\sin x)+\mathrm{c} \quad \ldots \text{(i)}$$
when $x=0, y=1$ ... [Given]

$\Rightarrow \mathrm{c}=2\log 2$

$\therefore$ (i) $\Rightarrow \log(y+1)=-\log(2+\sin x)+\log 4$

$$\Rightarrow \log(y+1)=\log\left(\frac{4}{2+\sin x}\right)$$
$$\Rightarrow y+1=\frac{4}{2+\sin x} \quad \text{... (ii)}$$
$$\therefore \quad \text{When } x=\frac{\pi}{2}, \text{(ii)} \Rightarrow y=\frac{4}{3}-1=\frac{1}{3}$$

Question 42

If $A = \begin{bmatrix} 2a & -3b \\ 3 & 2 \end{bmatrix}$ and $A \cdot \text{adj } A = AA^T$, then $2a + 3b$ is

Options:

A. -1

B. 1

C. 5

D. -5

Answer: C

Solution:

$$A = \begin{bmatrix} 2a & -3b \\ 3 & 2 \end{bmatrix}$$

$$A \cdot \text{adj } A = \begin{bmatrix} 2a & -3b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3b \\ -3 & 2a \end{bmatrix}$$

$$= \begin{bmatrix} 4a + 9b & 0 \\ 0 & 9b + 4a \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 2a & -3b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2a & 3 \\ -3b & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4a^2 + 9b^2 & 6a - 6b \\ 6a - 6b & 13 \end{bmatrix}$$

$$\therefore \quad A \cdot \text{adj } A = A \cdot A^T$$

$$\Rightarrow \begin{bmatrix} 4a + 9b & 0 \\ 0 & 4a + 9b \end{bmatrix} = \begin{bmatrix} 4a^2 + 9b^2 \\ 6a - 6b \end{bmatrix}$$

$$\Rightarrow a = b \text{ and } 4a + 9b = 13$$

$$\Rightarrow a = b = 1$$

$$\Rightarrow 2a + 3b = 5$$

Question 43

$\int\left(\frac{\tan\left(\frac{1}{x}\right)}{x}\right)^2 dx =$

Options:

A. $x - \tan x + \mathrm{c}$, where c is a constant of integration

B. $\frac{1}{x} - \tan\left(\frac{1}{x}\right) + \mathrm{c}$, where c is a constant of integration.

C. $\frac{1}{x} + \tan\left(\frac{1}{x}\right) + \mathrm{c}$, where c is a constant of integration.

D. $x + \tan x + \mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

$$\text{Let I} = \int\left(\frac{\tan\left(\frac{1}{x}\right)}{x}\right)^2 \mathrm{d}x$$

$$\text{Let } \frac{1}{x} = \mathrm{t} \Rightarrow \frac{1}{x^2}\,\mathrm{d}x = -\mathrm{dt}$$

$$\therefore \quad \mathrm{I} = -\int \tan^2 \mathrm{t}\,\mathrm{d}t$$

$$= \int\left(1 - \sec^2 t\right) dt$$

$$= \mathrm{t} - \tan \mathrm{t} + \mathrm{c}$$

$$= \frac{1}{x} - \tan\left(\frac{1}{x}\right) + c$$

Question 44

$\int \frac{1}{(x+2)(1+x)^2} dx$ **has the value**

Options:

A. $2\log\left(\frac{x+2}{x^2+1}\right) + 4\tan^{-1} x + \mathrm{c}$, where c is a constant of integration.

B. $\log\frac{x+2}{x^2+1} - 4\tan^{-1} x + \mathrm{c}$, where c is a constant of integration.

C. $\log \frac{(x+2)^2}{(x^2+1)} + 4\tan^{-1} x + c$, where c is a constant of integration.

D. None

Answer: D

Solution:

The question cannot be solved due to insufficient data.

Question 45

If two angles of $\triangle \mathrm{ABC}$ are $\frac{\pi}{4}$ and $\frac{\pi}{3}$, then the ratio of the smallest and greatest sides are

Options:

A. $(\sqrt{3}-1) : 1$

B. $\sqrt{3} : \sqrt{5}$

C. $\sqrt{2} : \sqrt{3}$

D. $(\sqrt{3}-1) : 4$

Answer: A

Solution:

Let three angles of the triangle be given as $\mathrm{A} = \frac{\pi}{4}$, $\mathrm{B} = \frac{\pi}{3}$ and $\mathrm{C} = \frac{\pi}{4} + \frac{\pi}{6}$

Let a, b, c be the opposite to angles A, B, C respectively.

As $\frac{\sin A}{a} = \frac{\sin C}{c}$, we get

Required Ratio $= \frac{a}{c} = \frac{\sin A}{\sin C}$

$= \frac{\sin\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$

$$= \frac{\sin\frac{\pi}{4}}{\sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}}$$
$$= \frac{2}{\sqrt{3}+1}$$
$$= \frac{2(\sqrt{3}-1)}{2}$$
$$= (\sqrt{3}-1) : 1$$

Question 46

In $\triangle \mathrm{ABC}, \mathrm{m}\angle\mathrm{B} = \frac{\pi}{3}$ and $\mathrm{m}\angle\mathrm{C} = \frac{\pi}{4}$. Let point D divide BC internally in the ratio $1 : 3$, then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ has the value

Options:

A. $\frac{1}{3}$

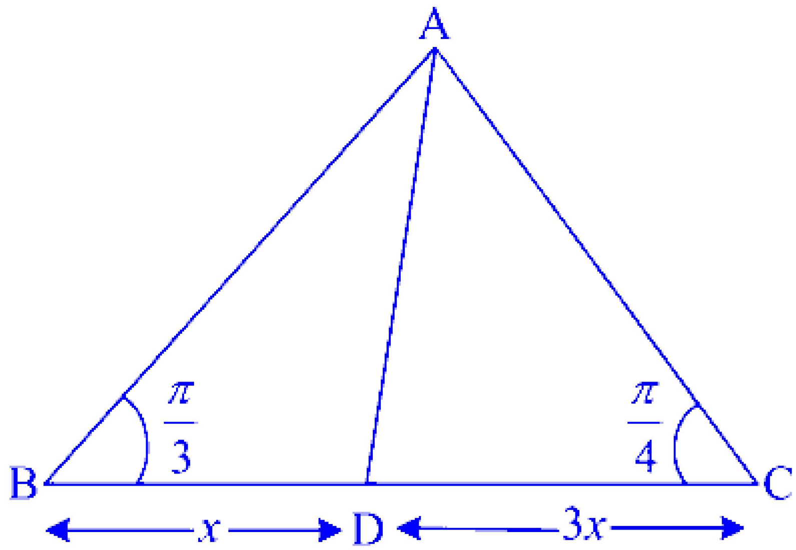
B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{6}}$

D. $\sqrt{\frac{2}{3}}$

Answer: C

Solution:



In $\triangle ABD$,

$$\frac{\sin(\angle BAD)}{BD} = \frac{\sin(\angle ABD)}{AD}$$

$$\Rightarrow \frac{\sin(\angle \mathrm{BAD})}{x} = \frac{\frac{\sqrt{3}}{2}}{\mathrm{AD}}$$

$$\Rightarrow \mathrm{AD} = \frac{\sqrt{3}x}{2\sin(\angle \mathrm{BAD})} \quad .... \text{(i)}$$

In $\triangle \mathrm{ADC}$,

$$\frac{\sin(\angle CAD)}{DC} = \frac{\sin(\angle ACD)}{AD}$$

$$\Rightarrow \frac{\sin(\angle CAD)}{3x} = \frac{\frac{1}{\sqrt{2}}}{\mathrm{AD}}$$

$$\therefore \quad \mathrm{AD} = \frac{3x}{\sqrt{2}\sin(\angle \mathrm{CAD})} \quad .... \text{(ii)}$$

From (i) and (ii), we get

$$\frac{\sqrt{3}x}{2\sin(\angle \mathrm{BAD})} = \frac{3x}{\sqrt{2}\sin(\angle \mathrm{CAD})}$$

$$\therefore \frac{\sin(\angle \mathrm{BAD})}{\sin(\angle \mathrm{CAD})} = \frac{\sqrt{6}}{6} = \frac{1}{\sqrt{6}}$$

Question 47

If $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$, then x is

Options:

A. 1

B. $\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2\sqrt{3}}$

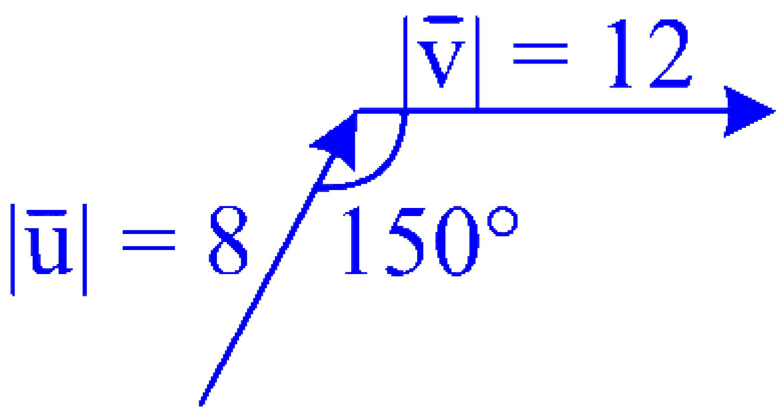
Answer: C

Solution:

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$$
$$\Rightarrow \tan^{-1}(1)-\tan^{-1}(x)=\frac{1}{2}\tan^{-1}x$$
$$\Rightarrow \frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$$
$$\Rightarrow x=\tan\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}$$

Question 48

If



then $|\vec{u}\times\vec{v}|$ is

Options:

A. 96

B. 80

C. 42

D. 48

Answer: D

Solution:

$$
\begin{aligned}
|\overline{\mathrm{u}} \times \overline{\mathrm{v}}| &= |\overline{\mathrm{u}}||\mathrm{v}| \sin (150^{\circ}) \\
&= 8 \times 12 \times \frac{1}{2} \\
&= 48
\end{aligned}
$$

Question 49

Three fair coins with faces numbered 1 and 0 are tossed simultaneously. Then variance (X) of the probability distribution of random variable X, where X is the sum of numbers on the upper most faces, is

Options:

A. 0.7

B. 0.75

C. 0.65

D. 0.6

Answer: B

Solution:

Possible value of ,X are $0, 1, 2, 3$

Here,

$$
\begin{aligned}
& S = \{000, 001, 010, 100, 111, 110, 101, 011\} \\
\therefore \quad & n(S) = 8
\end{aligned}
$$

x_i	0	1	2	3
p_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\therefore \quad \mathrm{E(X)} = \sum x_{\mathrm{i}} \mathrm{p_i} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8}$$

$$\mathrm{E}\left(\mathrm{X}^2\right) = \sum x_i^2 \mathrm{p_i^2} = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{24}{8}$$

$$\begin{aligned} \therefore \quad \text{Variance } &= \mathrm{E}\left(\mathrm{X}^2\right) - [\mathrm{E(X)}]^2 \\ &= 3 - \left(\frac{3}{2}\right)^2 \\ &= 3 - \frac{9}{4} \\ &= \frac{3}{4} \\ &= 0.75 \end{aligned}$$

Question 50

If $\mathrm{f}(x) = x^2 + 1$ and $\mathrm{g}(x) = \frac{1}{x}$, then the value of $\mathrm{f}(\mathrm{g}(\mathrm{g}(\mathrm{f}(x))))$ at $x = 1$ is

Options:

A. 4

B. 1

C. 5

D. 3

Answer: C

Solution:

$$f(x)=x^2+1,\ g(x)=\frac{1}{x}$$

$$\begin{aligned}\therefore\quad f(g(g(f(x)))) &= f\left(g\left(g\left(x^2+1\right)\right)\right)\\ &= f\left(g\left(\frac{1}{x^2+1}\right)\right)\\ &= f\left(x^2+1\right)\\ &= \left(x^2+1\right)^2+1\end{aligned}$$

$\therefore$ At $x=1$, we get the value of above function

$$\begin{aligned}&=\left[(1)^2+1\right]^2+1\\ &=5\end{aligned}$$

Question 51

Which of the following expressions represents molar conductivity of AB_3 type electrolyte?

Options:

A. $3\lambda^{\circ}_{A^{\cdots}} + \lambda^{\circ}_{B^{-}}$

B. $\lambda^{\circ}_{A^{\cdots}} + \lambda^{\circ}_{B^{-}}$

C. $\lambda^{\circ}_{A^{\cdots}} + 3\lambda^{\circ}_{B^{-}}$

D. $2\lambda^{\circ}_{A^{\cdots}} + \lambda^{\circ}_{B^{-}}$

Answer: C

Solution:

$\Lambda_0 = n_+\lambda^0_+ + n_-\lambda^0_-$

Where, λ_+ and λ_- are molar conductivities of cation and anion, respectively, and n_+ and n are the number of moles of cation and anion specified in the chemical formula of an electrolyte.

Question 52

What is the mass of $KClO_{3(s)}$ required to liberate 22. $4\ dm^3$ **oxygen at STP during thermal decomposition?**

(**Molar Mass of** $KClO_{3(s)} = 122.5\ g/mol$)

Options:

A. $122.5\ g$

B. 81.67 g

C. 10.25 g

D. 8.16 g

Answer: B

Solution:

$$\underset{[2\ \text{moles}]}{2KClO_3} \longrightarrow 2KCl + \underset{[3\ \text{moles}]}{3O_2\uparrow}$$

2 moles of $KClO_3 = 2 \times 122.5 = 245$ g

3 moles of O_2 at STP occupy $= (3 \times 22.4 dm^3)$

Thus, 245 g of potassium chlorate will liberate $67.2\ dm^3$ of oxygen gas.

Let 'x' gram of $KClO_3$ liberate $22.4\ dm^3$ of oxygen gas at S.T.P.

$\therefore \quad x = \frac{245 \times 22.4}{3 \times 22.4} = 81.67$ g

----

Question 53

Identify product B in the following reaction.

$$\text{Aniline} \xrightarrow[273\ K]{NaNO_2 + HCl} A \xrightarrow[\Delta]{H_2O} B + N_2\uparrow$$

Options:

A. Chlorobenzene

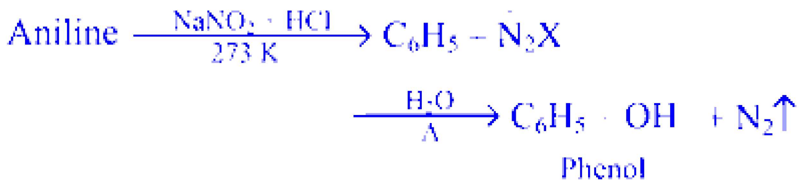
B. Benzyl alcohol

C. Benzenediazoniumchloride

D. Phenol

Answer: D

Solution:



Question 54

If $0.15\ \mathrm{m}$ aqueous solution of KCl freezes at $-0.511^{\circ}\mathrm{C}$, calculate van't Hoff factor of KCl (cryoscopic constant of water is $1.86\ \mathrm{K\ kg\ mol^{-1}}$)

Options:

A. 1.45

B. 1.26

C. 1.82

D. 3.00

Answer: C

Solution:

$$\Delta T_f = T_f^0 - T_f = 0 - (-0.51^{\circ}C) = 0.51^{\circ}C = 0.51\ K$$

$$\therefore \quad \Delta T_f = iK_f m$$

$$\therefore \quad i = \frac{\Delta T}{K_f m} = \frac{0.51\ K}{1.86\ K\ kg\ mol^{-1} \times 0.15\ mol\ kg^{-1}} = 1.82$$

Question 55

Which among the following is NOT the feature of reversible process?

Options:

A. The driving and opposing forces differ by large amount.

B. The process can be reversed by infinitesimal change in pressure.

C. A reversible process proceeds very slowly.

D. The system attains mechanical equilibrium at the end of every step.

Answer: A

Solution:

The correct answer is Option A, "The driving and opposing forces differ by large amount." To understand why, let's explore each option in relation to the characteristics of a reversible process:

Option A: The driving and opposing forces differ by large amount.

This is NOT a feature of a reversible process. In a reversible process, the change occurs in such a manner that the system and surroundings can be returned to their original states by exactly reversing the change. For this to happen, the driving force (such as pressure or temperature causing the change) must differ from the opposing force by an infinitesimally small amount, not a large one. This way, the system can always be in a state of equilibrium, or near equilibrium, with its surroundings.

Option B: The process can be reversed by infinitesimal change in pressure.

This is a feature of a reversible process. Since a reversible process is always close to equilibrium, only an infinitesimally small deviation is needed to reverse the direction of the process, whether it be a change in pressure, volume, or any other thermodynamic variable.

Option C: A reversible process proceeds very slowly.

This is also a feature of a reversible process. The process must proceed at an infinitely slow rate to ensure that the system undergoes a series of equilibrium states. This slow progression means that the system has time to adjust to virtually indistinguishable changes in the external conditions.

Option D: The system attains mechanical equilibrium at the end of every step.

This is another characteristic of a reversible process. Because the process happens so slowly and the driving forces are matched very closely by the opposing forces, the system can be said to be continually in mechanical (as well as thermal and chemical) equilibrium at each stage of the process.

Thus, the option that does not describe a reversible process is Option A, as a large difference between driving and opposing forces would lead to an irreversible process, where the system cannot be returned to its original state without causing changes in the surroundings.

Question 56

Which from following functional groups is at the lowest order to decide principal functional group in polyfunctional compound?

Options:

A. $-COOR$

B. $-COCI$

C. $> C = 0$

D. $-CONH_2$

Answer: C

Solution:

The principal functional group is decided on the basis of the following order of priority:

$$-COOH > -SO_3H > -COOR >$$

$$-COCl > -CONH_2 > -CN > -CHO >$$

$$>C=O > -OH > -NH_2 > \overset{|}{\underset{|}{C}} = \overset{|}{\underset{|}{C}} > -C \equiv C-$$

Question 57

Calculate the E_{cell} **for** $Zn_{(s)} \left| Zn^{++}_{(0.1M)} \right| \left| Cr^{+++}_{(0.1M)} \right| Cr_{(s)}$ **at** $25°C$ **if** E°_{cell} **is** 0.02 **V**

Options:

A. -0.05 V

B. 0.03 V

C. -0.06 V

D. 0.07 V

Answer: B

Solution:

The reaction is

$$3Zn_{(3)} + 2Cr^{3+}_{(0.1M)} \longrightarrow 3Zn^{2+}_{(0.1)} + 2Cr_{(s)}$$

$$E_{cell} = E^{o}_{cell} - \frac{0.0592 \text{ V}}{n} \log_{10} \frac{[Zn^{2+}]^3}{[Cr^{3+}]^2}$$

$$\begin{aligned} \therefore \quad E_{cell} &= 0.02 \text{ V} - \frac{0.0592 \text{ V}}{6} \log_{10} \frac{(0.1)^3}{(0.1)^2} \\ &= 0.02 \text{ V} - \frac{0.0592 \text{ V}}{6} \log_{10} 0.1 \\ &= 0.02 \text{ V} + \frac{0.0592 \text{ V}}{6} \times 1 \\ &= 0.02 \text{ V} + 0.0099 \text{ V} \\ &= 0.03 \text{ V} \end{aligned}$$

Question 58

Which from following formulae is of galena?

Options:

A. $CaSO_4 \cdot 2H_2O$

B. PbS

C. ZnS

D. $BaSO_4$

Answer: B

Solution:

Galena is a natural mineral form of lead sulfide. It is the most important ore of lead and an important source of silver. The chemical formula for galena is lead(II) sulfide, which is represented by the formula:

PbS

Based on the given options, the correct formula for galena is:

Option B: PbS

Here's a brief explanation of the other options provided:

- Option A: $CaSO_4 \cdot 2H_2O$ is the formula for gypsum, a mineral composed of calcium sulfate dihydrate.

- Option C: ZnS is the chemical formula for sphalerite, which is the main ore of zinc.

- Option D: $BaSO_4$ is the formula for barite, a mineral consisting of barium sulfate.

Therefore, Option B is the correct formula for galena.

Question 59

What is the solubility of $\mathrm{AgCl}_{(s)}$ if its solubility product is 1.6×10^{-10} ?

Options:

A. $1.26 \times 10^{-5}\mathrm{M}$

B. $1.00 \times 10^{-9}\mathrm{M}$

C. $2.6 \times 10^{-5}\mathrm{M}$

D. $1.56 \times 10^{-9}\mathrm{M}$

Answer: A

Solution:

$$\mathrm{AgCl}_{(s)} \rightleftharpoons \mathrm{Ag}^{+}_{(sq)} + \mathrm{Cl}^{-}_{(sq)}$$
$$x = 1, \mathrm{y} = 1$$
$$\mathrm{K}_{\mathrm{sp}} = x^{x}\mathrm{y}^{\mathrm{y}}\mathrm{S}^{x+\mathrm{y}} = (1)^{1}(1)^{1}\ \mathrm{S}^{1+1} = \mathrm{S}^{2}$$
$$\mathrm{S} = \sqrt{\mathrm{K}_{\mathrm{sp}}} = \sqrt{1.6 \times 10^{-10}}$$
$$= 1.26 \times 10^{-5}\mathrm{M}$$

Question 60

Which from following nanoparticle catalysts is used in photocatalysis?

Options:

A. TiO_2

B. Pd

C. Pt

D. Au

Answer: A

Solution:

Among the options provided, Option A TiO_2 (titanium dioxide) is commonly used as a nanoparticle catalyst in photocatalysis. Titanium dioxide is a well-known photocatalyst due to its strong oxidizing power, high photocatalytic efficiency, chemical stability, non-toxicity, and low cost. When it is irradiated with light at or above its band-gap energy (usually UV light), it can create electron-hole pairs, leading to the formation of reactive oxygen species capable of breaking down organic pollutants and disinfecting water.

While Options B (Pd, palladium), C (Pt, platinum), and D (Au, gold) can all be used as catalysts in various chemical reactions, they are not typically used for photocatalysis in the same way that TiO_2 is. These noble metals are more commonly associated with applications in oxidation, hydrogenation reactions, and as catalysts in fuel cells where they facilitate redox reactions without necessarily utilizing light as a reagent.

Question 61

A gas absorbs $150\ \mathrm{J}$ heat and expands by $300\ \mathrm{cm}^3$ against a constant external pressure $2 \times 10^5\ \mathrm{N\ m}^{-2}$, What is $\Delta \mathrm{U}$ of the system?

Options:

A. $210\ \mathrm{J}$

B. $90\ \mathrm{J}$

C. $450\ \mathrm{J}$

D. $-300\ \mathrm{J}$

Answer: B

Solution:

$300\ \mathrm{cm}^3 = 300 \times 10^{-6}\ \mathrm{m}^3$
$\mathrm{W} = -P_{\mathrm{ext}}\ \ \Delta\mathrm{V}$
$= -2 \times 10^5\ \mathrm{N\ m}^{-2} \times \left(300 \times 10^{-6}\ \mathrm{m}^3\right)$
$= -60\ \mathrm{N\ m}$
$= -60\ \mathrm{J}$
$\therefore\quad W = -60\ \mathrm{J}$

Now,

$$\begin{aligned}\therefore\quad \Delta\mathrm{U} &= \mathrm{q} + \mathrm{P}_{\mathrm{ex}}\Delta\mathrm{V} \\ &= 150\ \mathrm{J} + (-60\ \mathrm{J}) \\ &= 90\ \mathrm{J}\end{aligned}$$

Question 62

A first order reaction takes 23.03 minutes for 20% decomposition. Calculate its rate constant.

Options:

A. 5.6×10^{-3} minute^{-1}

B. 4.5×10^{-3} minute^{-1}

C. 6.5×10^{-3} minute^{-1}

D. 9.69×10^{-3} minute^{-1}

Answer: D

Solution:

$[A]_0 = 100\%, [A]_t = 100 - 20 = 80\%$

Substitution of these in above

$$k = \frac{2.303}{t}\log_{10}\frac{[A]_0}{[A]_t}$$

$$k = \frac{2.303}{t}\log_{10}\frac{100}{80}$$

$$= \frac{2.303}{23.03 \text{ min}}\log_{10}(1.25)$$

$$= \frac{2.303}{23.03 \text{ min}} \times 0.0969$$

$$= 9.69 \times 10^{-3} \text{ minute }^{-1}$$

Question 63

Which among the following is dicarboxylic acid?

Options:

A. Valeric acid

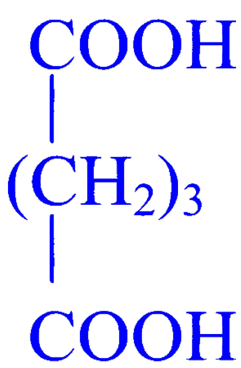
B. Caproic acid

C. Glutaric acid

D. Butyric acid

Answer: C

Solution:



Glutaric acid

Question 64

Which from following elements belongs to group 17 of periodic table?

Options:

A. At

B. Zn

C. As

D. Te

Answer: A

Solution:

Group 17 of the periodic table consists of elements known as halogens. The elements in Group 17 are fluorine (F), chlorine (Cl), bromine (Br), iodine (I), and astatine (At). They are located in the penultimate (second to last) column of the periodic table. Each of these elements has seven electrons in its outermost shell, which is the characteristic electron configuration of halogens.

Based on the options provided:

Option A: At (Astatine) - Astatine is indeed a halogen and belongs to Group 17 of the periodic table.

Option B: Zn (Zinc) - Zinc is a transition metal and belongs to Group 12, not Group 17.

Option C: As (Arsenic) - Arsenic is a metalloid and is located in Group 15, not Group 17.

Option D: Te (Tellurium) - Tellurium is a metalloid and is located in Group 16, just before the halogens.

Therefore, the correct answer is:

Option A: At (Astatine)

Question 65

What is the number of moles of donor atoms in n mole of NO_2^- ?

Options:

A. 3n

B. 0

C. 2n

D. n

Answer: C

Solution:

The ligand NO_2^- links to metal ion through nitrogen or oxygen. Hence, there are two donor atoms per molecule and 2n moles donor atoms per n mole.

Question 66

Calculate the density of an element having molar mass $27 \mathrm{~g} \mathrm{~mol}^{-1}$ that forms fcc unit cell. $\left[\mathrm{a}^3 \cdot \mathrm{N}_{\mathrm{A}}=38.5 \mathrm{~cm}^3 \mathrm{~mol}^{-1}\right]$

Options:

A. $2.8 \mathrm{~g} \mathrm{~cm}^{-3}$

B. $3.5 \mathrm{~g} \mathrm{~cm}^{-3}$

C. $2.1 \mathrm{~g} \mathrm{~cm}^{-3}$

D. $4.1 \mathrm{~g} \mathrm{~cm}^{-3}$

Answer: A

Solution:

Density, $\rho=\frac{M \mathrm{n}}{\mathrm{a}^3 \mathrm{~N}_{\mathrm{A}}}$

$\rho=\frac{27 \mathrm{~g} \mathrm{~mol}^{-1} \times 4 \text{ atom}}{38.5 \mathrm{~cm}^3 \mathrm{~mol}^{-1}}$

$=2.8 \mathrm{~g} \mathrm{~cm}^{-3}$

Question 67

Which of the following is not a globular protein?

Options:

A. Myosin

B. Insulin

C. Legumelin

D. Serum albumin

Answer: A

Solution:

Globular proteins are a common type of protein that are more or less soluble in aqueous solutions and have complex tertiary or quaternary structures. They typically carry out dynamic functions in organisms, such as catalyzing biochemical reactions (enzymes), transporting molecules (transport proteins), and regulating processes (hormones). Let's look at the options given:

- **Option A: Myosin -** Myosin is actually not a globular protein but rather a fibrous protein that functions in muscle contraction. It interacts with actin filaments in a process that converts chemical energy in the form of ATP to mechanical energy, thereby generating force and movement.

- **Option B: Insulin -** Insulin is a globular protein and a hormone that regulates glucose levels in the blood. It is produced by the pancreas and has a key role in the metabolism of carbohydrates, fats, and proteins.

- **Option C: Legumelin -** Legumelin is a globular storage protein found in seeds, such as those of legumes. It serves as a reserve of amino acids for the plant embryo during germination.

- **Option D: Serum albumin -** Serum albumin is the main protein in plasma and is also a globular protein. It has multiple roles including maintaining osmotic pressure, binding and transport of various substances in the blood.

Based on this information, the correct answer to the question—identifying the non-globular protein—would be:

Option A: Myosin

Question 68

Crotonyl alcohol is an example of

Options:

A. Allylic alcohol

B. Benzylic alcohol

C. Vinylic alcohol

D. Polyhydric alcohol

Answer: A

Solution:

Crotonyl alcohol ($H_3CCH = CHCH_2OH$) : Allylic alcohol

Question 69

What type of following solids the ice is

Options:

A. ionic solid

B. covalent network solid

C. molecular solid

D. metallic solid

Answer: C

Solution:

The correct option for the type of solid that ice represents is Option C, molecular solid.

Molecular solids are composed of atoms or molecules held together by intermolecular forces such as van der Waals forces, dipole-dipole interactions, or hydrogen bonds. Ice is the solid form of water (H_2O), and in ice, water molecules are held together by hydrogen bonds. These hydrogen bonds form a regular lattice that gives ice its solid structure at low temperatures. The molecules in ice do not share electrons as they would in a covalent network solid, nor do they form the ionic lattices seen in ionic solids. Additionally, ice does not have the electron "sea" characteristic of metallic solids where valence electrons are delocalized over a lattice of metal cations. Therefore, the best description for ice is that of a molecular solid, where discrete molecules are arranged in a specific geometrical pattern.

Question 70

A buffer solution is prepared by mixing 0.01 M weak acid and 0.05 M solution of a salt of weak acid and strong base. What is the pH of buffer solution? $(\text{pKa} = 4.74)$

Options:

A. 3.34

B. 4.80

C. 5.44

D. 6.93

Answer: C

Solution:

$$\begin{aligned} pH &= pK_a + \log_{10} \frac{[\text{ salt }]}{[\text{ acid}]} \\ &= 4.74 + \log_{10} \frac{0.05}{0.01} \\ &= 4.74 + 0.7 = 5.44 \end{aligned}$$

Question 71

Which of the following gases is formed during oxidation of trichloromethane?

Options:

A. $CO_{(g)}$

B. $CO_{2(g)}$

C. $COCl_{2(g)}$

D. $H_{2(g)}$

Answer: C

Solution:

Chloroform (trichloromethane) when exposed to air and light forms a poisonous compound phosgene ($COCl_2$) so it is stored in dark coloured air tight bottles.

Question 72

What type of ligand the EDTA is?

Options:

A. Monodentate

B. Bidentate

C. Tetradentate

D. Hexadentate

Answer: D

Solution:

EDTA, which stands for ethylenediaminetetraacetic acid, is a chelating agent that can bind to metal ions through multiple atoms. Each molecule of EDTA4- can bind to a single metal ion through six donor atoms: the two nitrogen atoms from the ethylenediamine part and the four oxygen atoms from the carboxylic acid groups (tetraacetate part).

These donor atoms allow EDTA to form complex structures with metal ions, effectively "grabbing" the metal at six points, which is why EDTA is termed a hexadentate ligand. In coordination chemistry, a hexadentate ligand is one that uses six pairs of electrons to form bonds with a metal ion. When EDTA forms these complexes, it typically results in an octahedral geometry around the metal ion.

So the correct answer to the type of ligand that EDTA is:

Option D: Hexadentate

Question 73

Which of the following compounds has difficulty in breaking of $C - X$ bond during nucleophilic substitution reaction?

Options:

A. o-Nitrochlorobenzene

B. p-Nitrochlorobenzene

C. m-Nitrochlorobenzene

D. 2,4,6-Trinitrochlorobenzene

Answer: C

Solution:

Greater the number of electron withdrawing groups at o/p position, greater is the reactivity towards nucleophilic substitution reaction. Electron withdrawing group at meta position has practically no effect on reactivity. Hence, among the given, m-nitrochlorobenzene has difficulty in breaking of C – X bond during nucleophilic substitution.

Question 74

What is the change in oxidation number of Cr in the following redox reaction?

$$3H_2O_{2(aq)} + Cr_2O_{7\,(aq)}^{2-} \quad + 8H_{(aq)}^{+} \longrightarrow 3O_{2(\,g)} + 2Cr^3 + 7H_2O$$

Options:

A. +2 to +3

B. −2 to +3

C. +7 to +3

D. +6 to +3

Answer: D

Solution:

$$\underset{+6}{\uparrow} Cr_2O_{7(aq)}^{2-} \longrightarrow 2\underset{+3}{\uparrow}Cr_{(aq)}^{3+}$$

Question 75

Equal masses of $H_{2(g)}$ and $He_{(g)}$ are enclosed in a container at constant temperature. The ratio of partial pressure of H_2 to He is

Options:

A. 1 : 1

B. 1 : 2

C. 2 : 1

D. 1 : 4

Answer: C

Solution:

Let the mass be ',x'.

$$n_{H_2} = \frac{xg}{2 \text{ g mol}^{-1}} = \frac{x}{2}$$

$$n_{He} = \frac{xg}{4 \text{ g mol}^{-1}} = \frac{x}{4}$$

$$n_{Total} = \frac{3x}{4}$$

Now,

$$x_{H_2} = \frac{n}{n_{Total}} = \frac{x/2}{3x/4} = \frac{2}{3}$$

$$x_{H_e} = \frac{n}{n_{Total}} = \frac{x/4}{3x/4} = \frac{1}{3}$$

Now,

$$P_{H_2} = x_{H_2} \times P_{Total} = \frac{2}{3} \times P$$

$$P_{He} = x_{He} \times P_{Total} = \frac{1}{3} \times P$$

$$P_{H_2} : P_{H_e} = \frac{2/3P}{1/3P} = 2 : 1$$

Question 76

Identify the reaction intermediate of following reaction.

$$\begin{array}{ll} \text{i.} & 2SO_{2(g)} + 2NO_{2(g)} \rightarrow 2SO_{3(g)} + 2NO \\ \text{ii.} & 2NO_{(g)} + O_{2(g)} \rightarrow 2NO_{2(g)} \\ \hline & 2SO_{2(g)} + O_{2(g)} \rightarrow 2SO_{3(g)} \end{array}$$

Options:

A. $NO_{2(g)}$

B. $NO_{(g)}$

C. $SO_{2(g)}$

D. $O_{2(g)}$

Answer: B

Solution:

Intermediate should not be present in overall reaction. $NO_{(g)}$ is the only species in the above reaction mechanism that is absent in overall reaction and hence, it is an intermediate.

Question 77

What is the number of $-OH$ groups present in one molecule of ribose?

Options:

A. 2

B. 3

C. 4

D. 5

Answer: C

Solution:

CHO
|
$(CHOH)_3$
|
CH_2OH

Ribose

Question 78

What is the position of copper in long form of periodic table?

Options:

A. Group -12, period-3

B. Group-11, period-4

C. Group-8, period-4

D. Group-11, period-5

Answer: B

Solution:

The correct position of copper in the long form of the periodic table is given by Option B: Group-11, period-4.

In the periodic table, copper has the atomic number 29. It belongs to the group of elements known as the transition metals. Specifically, copper is in the 11th group, which is sometimes referred to as group IB in older notations. It is in the 4th period, which indicates that it has four electron shells or energy levels. Thus, when properly identifying copper's place in the periodic table, we should say it is located in Group 11, Period 4.

The other options can be dismissed as follows:

Option A (Group-12, period-3) - This is incorrect because Group 12 elements include zinc (Zn), cadmium (Cd), and mercury (Hg), and Period 3 elements do not include transition metals.

Option C (Group-8, period-4) - This is incorrect because Group 8 elements include iron (Fe), ruthenium (Ru), and osmium (Os).

Option D (Group-11, period-5) - This is incorrect because Period 5 of Group 11 contains silver (Ag), while copper is in Period 4.

Question 79

What is bond order of F_2 molecule?

Options:

A. $\frac{1}{2}$

B. 1

C. 2

D. 3

Answer: B

Solution:

The electronic configuration of F_2

$(\sigma 1s)^2(\sigma^* 1\ s)^2(\sigma 2s)^2(\sigma^* 2\ s)^2(\sigma 2p_z)^2(\pi 2p_x)^2(\pi 2p_y)^2(\pi^* 2p_x)^2(\pi^* 2p_y)^2$

Bond order of F_2 molecule $= \dfrac{N_b - N_a}{2} = \dfrac{10 - 8}{2} = 1$

Question 80

Which of the following compounds does NOT develop intermolecular hydrogen bonding?

Options:

A. Cyclohexylamine

B. Allylamine

C. Trimethylamine

D. Diphenylamine

Answer: C

Solution:

Trimethylamine, $N(CH_3)$ does not contain $N-H$ bond and hence it does not develop intermolecular hydrogen bonding.

Question 81

Which of the following is CORRECT IUPAC name of catechol?

Options:

A. Benzene-1,2-diol

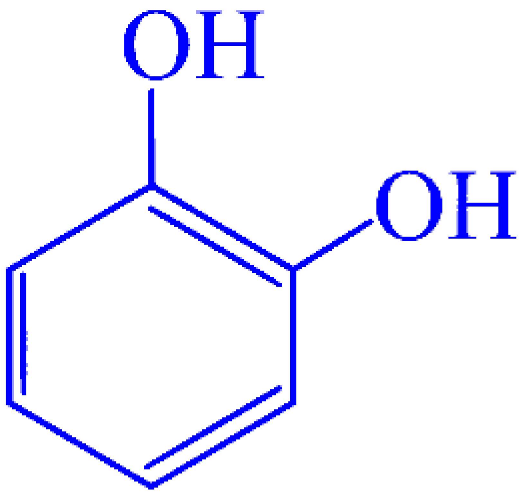
B. Benzene-1,3-diol

C. Benzene-1,4-diol

D. Benzene-1,3,5 triol

Answer: A

Solution:



Catechol
(Benzene -1,2-diol)

Question 82

What is the solubility of gas in water at $25^{\circ}\mathrm{C}$ if partial pressure is 0.346 bar [Henry's law constant is $[0.159\ \mathrm{mol\ dm^{-3}\ bar^{-1}}]$?

Options:

A. $0.055\ \mathrm{mol\ dm^{-3}}$

B. $0.028\ \mathrm{mol\ dm^{-3}}$

C. $0.083\ \mathrm{mol\ dm^{-3}}$

D. $0.11\ \mathrm{mol\ dm^{-3}}$

Answer: A

Solution:

$$S = K_H P = 0.159 \text{ mol dm}^{-3} \text{ bar}^{-1} \times 0.346 \text{ bar}$$
$$= 0.055 \text{ mol dm}^{-3}$$

Question 83

Which from following coloured light has the highest energy?

Options:

A. Red

B. Blue

C. Yellow

D. Violet

Answer: D

Solution:

Shorter the wavelength, larger is the frequency, and higher is the energy. Violet light has the shortest wavelength (400 nm).

Question 84

Which among the following is haloalkyne?

Options:

A. $CH_3 - CH_2 - CH = CH - X$

B. $CH_3 - C = C - CH_2 - X$

C. $CH \equiv C - CH_2 - CH_2 - X$

D. $CH_3 - CH_2 - C \equiv C - X$

Answer: D

Solution:

Haloalkynes are compounds that contain a halogen atom (such as fluoride, chlorine, bromine, or iodine) directly bonded to a carbon atom that is part of a carbon-carbon triple bond. In other words, for a compound to be classified as a haloalkyne, it must have a functional group that consists of both a carbon-carbon triple bond (alkyne part) and a halogen atom attached to one of the carbon atoms involved in the triple bond.

Now, let's examine the provided options:

Option A: $CH_3 - CH_2 - CH = CH - X$

This is not a haloalkyne because it has a carbon-carbon double bond rather than a triple bond, so it would be classified as a haloalkene.

Option B: $CH_3 - C = C - CH_2 - X$

Although this compound contains a carbon-carbon triple bond, the halogen is not attached to one of the carbons participating in that triple bond. Thus, this compound is not a haloalkyne either.

Option C: $CH \equiv C - CH_2 - CH_2 - X$

Similar to Option B, this compound has a carbon-carbon triple bond and a halogen, but the halogen is not directly attached to the carbon-carbon triple bond, so this is also not a haloalkyne.

Option D: $CH_3 - CH_2 - C \equiv C - X$

This compound has a carbon-carbon triple bond and a halogen atom directly bonded to one of the carbon atoms of the triple bond, making it a haloalkyne.

Therefore, the correct answer is:

Option D: $CH_3 - CH_2 - C \equiv C - X$

It is the haloalkyne among the given options because it has the halogen atom (X) directly connected to a carbon that is part of an alkyne group.

Question 85

Which element from the following possesses half-filled d-orbitals either in expected or in observed electronic configuration?

Options:

A. Fe

B. Mn

C. Ni

D. Co

Answer: B

Solution:

Mn (Observed and expected electronic configurations): $[Ar] 3 d^5 4 s^2$

Question 86

What is the radius of the fourth orbit of hydrogen atom?

Options:

A. 846.4 pm

B. 211.6 pm

C. 476.1 pm

D. 1322.5 pm

Answer: A

Solution:

$r_n = \frac{52.9(n)^2}{Z}$ pm

For hydrogen, $Z = 1$

$r_n = \frac{52.9 \times 4^2}{1} = 846.4$ pm

Question 87

A conductivity cell containing 5×10^{-4} M NaCl solution develops resistance 14000 ohms at 25°C. Calculate the conductivity of solution

if the cell constant is $0.84\ \mathrm{cm}^{-1}$

Options:

A. $6.0 \times 10^{-5} \Omega^{-1}\ \mathrm{cm}^{-1}$

B. $3.0 \times 10^{-5} \Omega^{-1}\ \mathrm{cm}^{-1}$

C. $9.0 \times 10^{-5} \Omega^{-1}\ \mathrm{cm}^{-1}$

D. $12.0 \times 10^{-5} \Omega^{-1}\ \mathrm{cm}^{-1}$

Answer: A

Solution:

$$\begin{aligned} k &= \frac{\text{cell constant}}{R} \\ &= \frac{0.84\ \mathrm{cm}^{-1}}{14000 \Omega} \\ &= 6.0 \times 10^{-5} \Omega^{-1}\ \mathrm{cm}^{-1} \end{aligned}$$

Question 88

Which among the following statements is NOT true for LDP?

Options:

A. It needs O_2 or peroxide as initiator synthesis process.

B. It is a branched chain polymer.

C. It needs low pressure about 6-7 atm in synthesis process.

D. It has low melting point than HDP.

Answer: C

Solution:

LDP is prepared by polymerization of ethylene under high pressure $(1000 - 2000 \text{ atm})$ and temperature $(350 - 570 \text{ K})$ in presence of traces of O_2 or peroxide as initiator.

Question 89

Equal masses in grams of H_2, N_2, Cl_2, and O_2, are enclosed in cylinders separately. If these gases expand isothermally and reversibly by 10 dm^3 at 300 K, the work done by gas is maximum for

Options:

A. H_2

B. N_2

C. Cl_2

D. O_2

Answer: A

Solution:

$W_{max} = -2.303 \text{ nRT } \log_{10} \frac{V_2}{V_1}$

Hence, $W_{max} \propto n$ (Given: $R, T, V_2, V_1 = \text{Constant}$)

$\therefore W_{max} \propto \frac{1}{M.W.}$ (Given: equal mass)

Hence, lower the molecular mass, greater is the work done. Among the given, H_2 has the lowest molecular mass.

Question 90

Which of the following compounds is obtained by Rosenmund reduction of benzoyl chloride?

Options:

A. Benzene

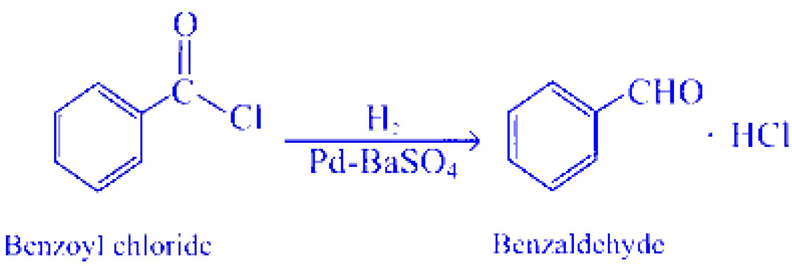
B. Benzyl alcohol

C. Benzaldehyde

D. Chlorobenzene

Answer: C

Solution:



Question 91

The partial vapour pressure of any volatile component of a solution is equal to the vapour pressure of the pure component multiplied by its mole fraction in the solution is called

Options:

A. Dalton's law

B. Avogadro's law

C. Raoult's law

D. Henry's law

Answer: C

Solution:

The statement that "The partial vapor pressure of any volatile component of a solution is equal to the vapor pressure of the pure component multiplied by its mole fraction in the solution" is known as Raoult's law. Therefore, the correct answer is:

Option C Raoult's law

Raoult's law can be mathematically expressed as:

$P_A = P_A^* \cdot X_A$

Where:

- P_A = partial vapor pressure of component A in the solution
- P_A^* = vapor pressure of the pure component A
- X_A = mole fraction of component A in the solution

This law applies to ideal solutions, where the interactions between molecules of different components are nearly the same as the interactions between molecules of the same components. In such mixtures, the presence of other components does not affect the vapor pressure of any given component significantly, other than through the mole fraction. Actual solutions may deviate from Raoult's law, especially at higher concentrations or with components that interact strongly with each other.

Question 92

Which from following polymers is grouped in the category of elastomers?

Options:

A. Neoprene

B. Terylene

C. Polystyrene

D. Bakelite

Answer: A

Solution:

Elastomers are a type of polymer that exhibit rubber-like elasticity, meaning they can be stretched to significant extents and yet return to their original shape upon release of the stress. Among the options provided, Neoprene is categorized as an elastomer.

Option A: Neoprene - Neoprene, also known as polychloroprene, is indeed an elastomer. It is synthetically produced and displays good chemical stability and maintains flexibility over a wide temperature range. Because of these properties, it is used in a variety of applications, such as in wetsuits, hoses, and as a material for sealants and adhesives.

Option B: Terylene - Terylene, which is better known as Polyester or specifically polyethylene terephthalate (PET), is not an elastomer but rather a thermoplastic polyester. It is used for making fibers for clothing, containers for liquids and foods, and in thermoforming applications.

Option C: Polystyrene - Polystyrene is a thermoplastic polymer, not an elastomer. It is hard and brittle and is commonly used in products like disposable cutlery, plastic models, CD and DVD cases, and in packaging.

Option D: Bakelite - Bakelite is a thermosetting phenol-formaldehyde resin, not an elastomer. It is one of the first synthetic plastics and is known for its electrical nonconductivity and heat-resistant properties, which make it suitable for use in electrical insulators and early plastic ware.

Therefore, the correct answer is Option A: Neoprene.

Question 93

Which among the following is NOT an example of salt of strong acid and weak base?

Options:

A. NH_4Cl

B. NH_4NO_3

C. $CuSO_4$

D. Na_2SO_4

Answer: D

Solution:

Na_2SO_4 is a salt of strong base (NaOH) and strong acid (H_2SO_4).

Question 94

Calculate the edge length of simple cubic unit cell if radius of an atom is 167.3 pm.

Options:

A. 473.2 pm

B. 334.6 pm

C. 386.3 pm

D. 836.5 pm

Answer: B

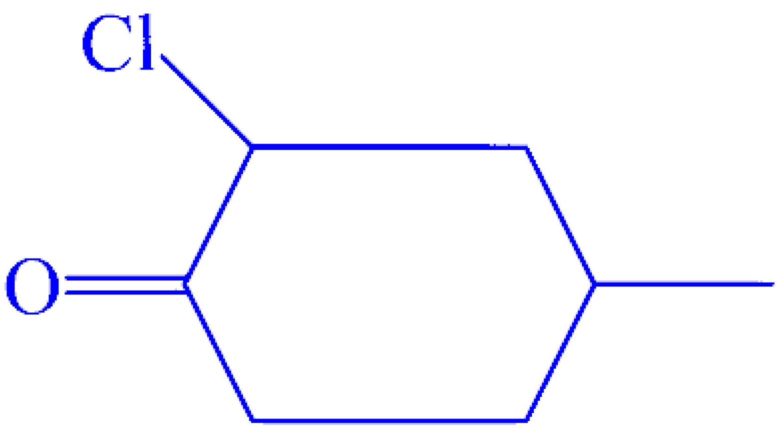
Solution:

For simple cubic unit cell, $a = 2r = 2 \times 167.3 \text{ pm} = 334.6 \text{ pm}$

For simple cubic unit cell, edge length is twice the radius of an atom.

Question 95

What is IUPAC name of following compound?



Options:

A. 1-Chloro-4-methylcyclohexan-2-one

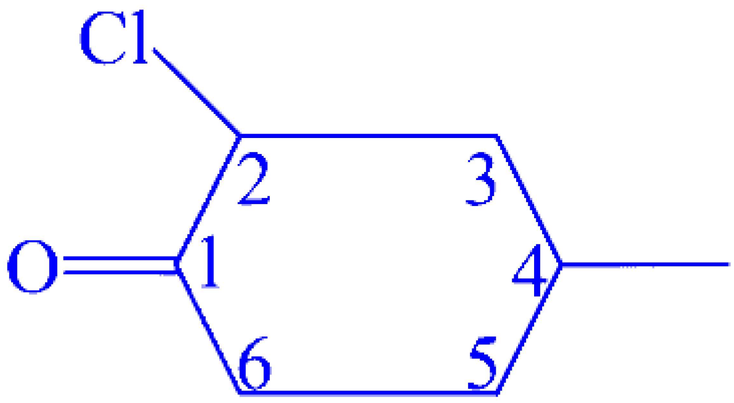
B. 2-Chloro-3-methyclohexanone

C. 2-Chloro-4-methylcyclohexanone

D. 4-Methyl-2-chlorocyclohexanone

Answer: C

Solution:



2-chloro-4-methylcyclohexanone

Question 96

Which element from following is used for cancer treatment?

Options:

A. Ba

B. Sr

C. Ra

D. Ru

Answer: C

Solution:

The element used for cancer treatment from the options listed is Option C, Ra, which stands for Radium. Radium is a radioactive element that has been historically used in various forms of cancer treatment, particularly before the advent of more modern techniques. It emits alpha particles that can kill cancer cells when placed near or within tumors. The most commonly known use of radium in medicine was in the form of radium needles, which were inserted into tumors to deliver a high radiation dose directly to the cancerous tissue. Although radium has largely been replaced by more targeted and less harmful forms of radiation therapy, its role in the history of cancer treatment is significant.

Here are the details for each of the options:

- Option A: Ba - Barium is not typically used for cancer treatment. However, barium compounds are used in medical imaging to enhance X-ray and CT imaging.

- Option B: Sr - Strontium, especially the radioactive isotope Strontium-89 ($^{89}_{38}Sr$), is used in the treatment of metastatic bone cancer. It acts in a manner similar to calcium and selectively localizes in the bone, providing targeted radiation therapy.

- Option C: Ra - Radium, particularly Radium-223 ($^{223}_{88}Ra$), is used in the treatment of prostate cancer that has spread to the bones.

- Option D: Ru - Ruthenium is not commonly used for cancer treatment; it's primarily utilized in catalysts and electrical contacts.

In the context of these options, while Radium and Strontium have applications in treating cancer, specifically Radium (Ra) is the more historically notable element for cancer therapy.

Question 97

The rate law for the reaction $\mathrm{A}+\mathrm{B} \rightarrow \mathrm{C}$ at $25^{\circ}\mathrm{C}$ is given by rate $= k[A][B]^2$. Calculate the rate of reaction if rate constant at same temperature is $6.25\ \mathrm{mol}^{-2}\ \mathrm{dm}^6\ \mathrm{s}^{-1}[[\mathrm{~A}]=1\mathrm{M},[\mathrm{B}]=0.2\mathrm{M}]$

Options:

A. $0.25\ \mathrm{mol\ dm}^{-3}\ \mathrm{s}^{-1}$

B. $0.5\ \mathrm{mol\ dm}^{-3}\ \mathrm{s}^{-1}$

C. $0.75\ \mathrm{mol\ dm}^{-3}\ \mathrm{s}^{-1}$

D. $1.25\ \mathrm{mol\ dm}^{-3}\ \mathrm{s}^{-1}$

Answer: A

Solution:

$$\begin{aligned}
\text{Rate } & = k[A][B]^2 \\
& = 6.25\ \mathrm{mol}^{-2}\ \mathrm{dm}^6\ \mathrm{s}^{-1} \times 1\ \mathrm{mol\ dm}^{-3} \\
& \quad \times 0.2\ \mathrm{mol\ dm}^{-3} \times 0.2\ \mathrm{mol\ dm}^{-3} \\
& = 0.25\ \mathrm{mol\ dm}^{-3}\ \mathrm{s}^{-1}
\end{aligned}$$

Question 98

Select the CORRECT increasing order of boiling points of alcohols, amines and carboxylic acids of comparable molar mass from the following.

Options:

A. Alcohols < Amines < Carboxylic acids

B. Amines < Carboxylic acids < Alcohols

C. Amines < Alcohols < Carboxylic acids

D. Carboxylic acids < Alcohols < Amines

Answer: C

Solution:

$N - H$ bonds in amines are less polar than $O - H$ bond in alcohols. Carboxyl group $(-COOH)$ of carboxylic acid contains $O - H$ bond which is responsible for formation of hydrogen bonding. Strength of hydrogen bonding in carboxylic acid is greater than that of alcohols.

The $O - H$ bond in carboxylic acid is polarized to a great extent as compared to alcohol. Hence, they have higher boiling points than corresponding alcohols.

Question 99

What is the number of moles of 'C' atoms present in n mole molecule of alkane if it exhibits three structural isomers?

Options:

A. 3n

B. 4n

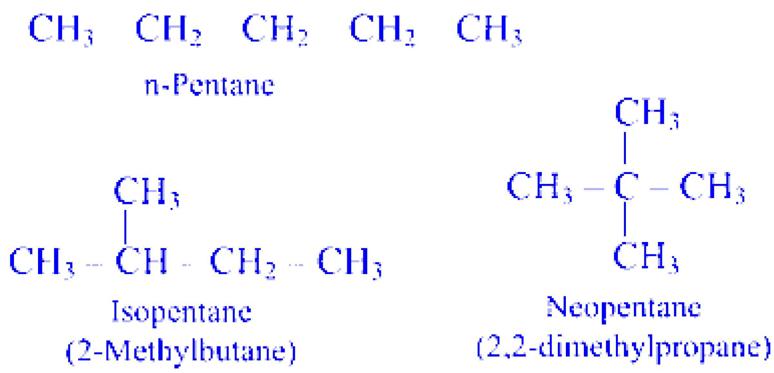
C. 5n

D. 6n

Answer: C

Solution:

Pentane (C_5H_{12}) exists in three isomeric forms:



Question 100

Which of the following reactions is used for the conversion of alkyl chloride to alkyl iodide?

Options:

A. Fitting reaction

B. Friedel Crafts reaction

C. Finkelstein reaction

D. Swartz reaction

Answer: C

Solution:

The conversion of alkyl chloride to alkyl iodide is typically done through the Finkelstein reaction, which involves the exchange of halide ions. Therefore, the correct answer is:

Option C: Finkelstein reaction

The Finkelstein reaction is an SN2 nucleophilic substitution reaction where an alkyl halide (usually a chloride or bromide) reacts with a sodium halide salt (usually sodium iodide) in acetone to form an alkyl iodide. The mechanism of the reaction ensures that the iodide ion is a better nucleophile in acetone and displaces the chloride or bromide ion. The reaction can be represented as:

$$R - Cl + NaI \rightarrow R - I + NaCl$$

Acetone is used as a solvent because it helps in the swift precipitation of the sodium chloride formed as a byproduct, driving the equilibrium towards the product side due to Le Chatelier's principle.

As for the other options:

Option A: The Fittig reaction is used to couple two aryl or vinyl halides in the presence of a metal, typically sodium.

Option B: The Friedel-Crafts reaction is a type of alkylation or acylation reaction where an alkyl or acyl group is introduced into an aromatic ring with the help of a Lewis acid catalyst like aluminium chloride, $AlCl_3$.

Option D: The Swarts reaction is another halogen exchange reaction primarily used for the conversion of alkyl chlorides or bromides to alkyl fluorides using metal fluoride salts such as AgF, Hg_2F_2, CoF_3, or SbF_3.

Question 101

A glass prism deviates the red and violet rays through 9° and 11° respectively. A second prism of equal angle deviates them through 11° and 13° respectively. The ratio of dispersive power of second prism to first prism is

Options:

A. 5 : 6

B. 6 : 5

C. 9 : 13

D. 13 : 9

Answer: A

Solution:

$\therefore$ Dispersive Power of Prism:

$$\omega = \frac{\delta_v - \delta_r}{\delta_y}$$
$$\delta_y = \frac{\delta_v + \delta_r}{2} = \frac{11+9}{2}$$

$\therefore$ For first Prism,

$$\omega_1 = \frac{2(11-9)}{11+9}$$
$$= \frac{1}{5}$$

For Second Prism:

$$\omega_2 = \frac{2(13-11)}{13+11}$$
$$= \frac{1}{6}$$

Ratio:

$$\frac{\omega_2}{\omega_1} = \frac{\frac{1}{6}}{\frac{1}{5}} = \frac{5}{6}$$

$\therefore$ The ratio is 5 : 6.

Question 102

Eight small drops of mercury each of radius 'r', coalesce to form a large single drop. The ratio of total surface energy before and after the change is

Options:

A. 2 : 1

B. 1 : 1

C. 1 : 4

D. 1 : 8

Answer: A

Solution:

Let R be the radius of the coalesced drop.

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$
$$R^3 = 8\pi r^3$$
$$R = 2r$$

Surface Energy $\mathrm{E} = \mathrm{T}.\,\mathrm{dA}$

$$\mathrm{E}_1 = 8 \times \mathrm{T} \times \mathrm{dA} = 8 \times \mathrm{T} \times 4\pi \mathrm{r}^2 \ldots. \text{ (i)}$$
$$\mathrm{E}_2 = \mathrm{T} \times \mathrm{dA} = \mathrm{T} \times 4\pi \mathrm{R}^2 \quad \text{.... (ii)}$$

Where E_1 and E_2 are the surface energies before and after coalescing.

Dividing equation (i) by (ii),

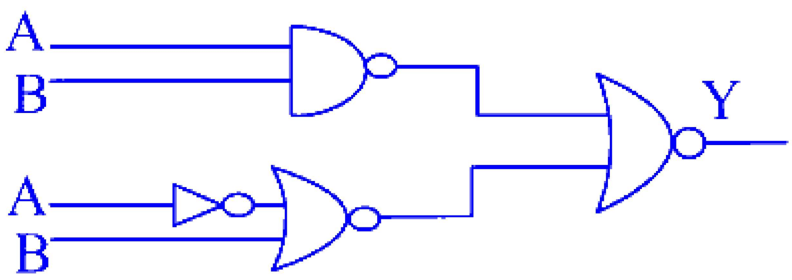
$$\frac{E_1}{E_2}=\frac{8r^2}{R^2}$$

but $\quad R=2r$

$$\therefore \quad \frac{E_1}{E_2}=\frac{8r^2}{4r^2}=\frac{2}{1}$$

Question 103

In the following digital logic circuit, the output Y will be '1' for inputs



Options:

A. A = 0, B = 0

B. A = 0, B = 1

C. A = 1, B = 0

D. A = 1, B = 1

Answer: D

Solution:

$\therefore$ There are two NOR gates, one NOT gate, and one NAND gate.

$\therefore$ Output of NAND gate: $\overline{A \cdot B}$

Output of NOT and NOR Gate: $\overline{\bar{A}+B}$

Final output: $\overline{(\overline{\mathrm{A} \cdot \mathrm{B}})+(\overline{\overline{\mathrm{A}}+\mathrm{B}})}$

So, the output Y is 1 only if the input A and B is 1.

$A = 1$
$B = 1$
$Y = \overline{\overline{1 \cdot 1} + \overline{1} + 1}$
$Y = 1$

Question 104

Two different radioactive elements with half lives 'T_1' and 'T_2' have undecayed atoms 'N_1' and 'N_2' respectively present at a given instant. The ratio of their activities at that instant is

Options:

A. $\frac{N_1 T_1}{N_2 T_2}$

B. $\frac{N_2 T_2}{N_1 T_1}$

C. $\frac{N_1 T_2}{N_2 T_1}$

D. $\frac{N_1 N_2}{T_1 T_2}$

Answer: C

Solution:

$\therefore$ Activity is given as:

$\therefore$ $A = \lambda N$

$$\lambda = \frac{\ln 2}{T}$$

$\therefore$ Ratio of two different radioactive elements will be:

$$\frac{A_1}{A_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2}$$
$$\frac{A_1}{A_2} = \frac{\frac{\ln 2}{T_1} N_1}{\frac{\ln 2}{T_2} N_2}$$
$$\frac{A_1}{A_2} = \frac{N_1 T_2}{N_2 T_1}$$

Question 105

The equation of wave motion is $Y=5\sin(10\pi t-0.02\pi x+\pi/3)$ where x is in metre and t in second. The velocity of the wave is

Options:

A. 300 m/s

B. 400 m/s

C. 500 m/s

D. 600 m/s

Answer: C

Solution:

$\therefore$ Equation of Wave: $\mathrm{y}=\mathrm{A}\sin(\mathrm{kx}-\omega\mathrm{t}+\phi)$

Value of k :

$$k=\frac{2\pi}{\lambda}$$
$$k=0.02\pi$$
$$\omega=10\pi$$

$\therefore$ Velocity of wave is $v=\frac{\omega}{k}$

$$v=\frac{10\pi}{0.02\pi}$$
$$v=500\ \mathrm{m/s}$$

Question 106

A potentiometer wire of length $4\ \mathrm{m}$ and resistance $5\ \Omega$ is connected in series with a resistance of $992\ \Omega$ and a cell of e.m.f. $4\ \mathrm{V}$ with internal resistance $3\ \Omega$. The length of $0.75\ \mathrm{m}$ on potentiometer wire balances the e.m.f. of

Options:

A. 4.00 mV

B. 3.75 mV

C. 3.00 mV

D. 2.50 mV

Answer: B

Solution:

$\therefore$ Total Resistance:

$R = 992 + 5 + 3 = 1000\Omega$

Voltage across 4 m wire:

$\frac{5}{995+5} \times 4 = 0.02\ \mathrm{V}$

$\therefore$ For one metre wire:

$\frac{0.02}{4} = 0.005\ \mathrm{V}$

$\therefore$ For 0.75 m wire:

$$\begin{aligned} 0.004 \times 0.75 &= 0.00375 \\ &= 3.75\ \mathrm{mV} \end{aligned}$$

Question 107

Select the correct statement from the following.

Options:

A. Gravitational force is stronger than electrostatic force

B. Gravitational as well as electrostatic force always attractive.

C. Gravitational as well as electrostatic force always act along the line joining the two objects.

D. Inverse square law $\left(F \propto \frac{1}{r^2}\right)$ is not obeyed by electrostatic force.

Answer: C

Solution:

The correct option is:

Option C: Gravitational as well as electrostatic force always act along the line joining the two objects.

Explanation:

Option A is incorrect because gravitational force is actually much weaker than the electrostatic force. The gravitational force between two particles is given by Newton's law of gravitation:

$$F_{\text{gravity}} = G\frac{m_1 m_2}{r^2}$$

where G is the gravitational constant, m_1 and m_2 are the masses of the particles, and r is the distance between their centers. On the other hand, the electrostatic force is given by Coulomb's law:

$$F_{\text{electrostatic}} = k\frac{|q_1 q_2|}{r^2}$$

where k is Coulomb's constant, and q_1 and q_2 are the electric charges of the particles. The electrostatic force is much stronger than the gravitational force because the gravitational constant G is a much smaller number than Coulomb's constant k.

Option B is incorrect because the electrostatic force can be either attractive or repulsive, depending on whether the charges are opposite (attractive) or like charges (repulsive). However, the gravitational force is always attractive, as it always acts to pull masses together.

Option C is correct. Both gravitational and electrostatic forces act along the straight line joining the centers of the two masses or charges, respectively. This line is known as the line of action for these forces.

Option D is incorrect because the electrostatic force does indeed obey an inverse square law, just like the gravitational force. Both forces diminish in strength with the square of the distance between the two objects, as is reflected in the equations for both forces (Newton's law of gravitation and Coulomb's law).

Question 108

End correction at open end for air column in a pipe of length 'l' is 'e'. For its second overtone of an open pipe, the wavelength of the wave is

Options:

A. $\frac{2(l+e)}{3}$

B. $\frac{2(l+2e)}{3}$

C. $\frac{4(l+e)}{5}$

D. $\frac{4(l+2e)}{5}$

Answer: B

Solution:

$$\therefore \quad n_3 = \frac{3v}{2l} = \frac{3v}{2(l+2e)}$$
$$\lambda_3 = \frac{2(l+2e)}{3} \quad \ldots.(\because v = n_3\lambda)$$

Question 109

In a stationary lift, time period of a simple pendulum is 'T'. The lift starts accelerating downwards with acceleration $\left(\frac{g}{4}\right)$, then the time period of the pendulum will be

Options:

A. $\frac{\sqrt{3}}{2}$ T

B. $\frac{2}{\sqrt{3}}$ T

C. $\frac{3}{4}$ T

D. $\frac{4}{3}$ T

Answer: B

Solution:

Time period of pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

When lift is accelerated downward with acceleration $\frac{g}{4}$,

$$g = g - \frac{g}{4}$$
$$g = \frac{3g}{4}$$

$\therefore$ New Time period will be

$$T_1 = 2\pi\sqrt{\frac{4L}{3g}}$$
$$T_1 = 2\pi\frac{2}{\sqrt{3}}\sqrt{\frac{L}{g}}$$
$$T_1 = \frac{2}{\sqrt{3}}T$$

Question 110

A body is projected vertically upwards from earth's surface of radius 'R' with velocity equal to $\frac{1^{\text{rd}}}{3}$ of escape velocity. The maximum height reached by the body is

Options:

A. $\frac{R}{8}$

B. $\frac{R}{6}$

C. $\frac{R}{4}$

D. $\frac{R}{9}$

Answer: A

Solution:

Δ K.E. $= \Delta$U

Let mass of the particle be M and that of the Earth be M_e

$\therefore \quad \frac{1}{2}Mv^2 = GM_eM\left(\frac{1}{R} - \frac{1}{R+h}\right)$ ..... (i)

Also, $g = \frac{GM_e}{R^2}$ ..... (ii)

Equation (i) can be written as,

$$\frac{1}{2}v^2 = G_e\left[\frac{R+h-R}{R(R+h)}\right] = \frac{G_e}{R^2}\left[\frac{Rh}{(R+h)}\right]$$

$$\therefore \quad \frac{1}{2}\left(\frac{1}{3}v_e\right)^2 = \frac{gRh}{R+h}$$

$$\therefore \quad \frac{1}{2}\left(\frac{1}{3}\sqrt{2gR}\right)^2 = \frac{gRh}{R+h}$$

$$\therefore \quad \frac{1}{2}\times\frac{1}{9}(2gR) = \frac{gRh}{R+h}$$

$$\therefore \quad \frac{h}{R+h} = \frac{1}{9}$$

$$\therefore \quad 9h = R+h$$

$$\therefore \quad 8h = R$$

$$\therefore \quad h = \frac{R}{8}$$

Given $\mathrm{V} = \frac{\mathrm{Ve}}{3}$

$$\therefore \quad \mathrm{h} = \frac{\mathrm{R}}{\left[\frac{\mathrm{V_e}}{\mathrm{V_e}/3}\right]^2 - 1} = \frac{\mathrm{R}}{9-1} = \frac{\mathrm{R}}{8}$$

Question 111

Which one of the following statements is WRONG regarding LED?

Options:

A. LEDs are energy efficient

B. LEDs have long time if properly manufactured.

C. Brightness of light emitted by LED cannot be controlled.

D. Colours produced by LED do not fade out.

Answer: C

Solution:

The statement that is WRONG regarding LED is:

Option C: Brightness of light emitted by LED cannot be controlled.

This statement is incorrect because the brightness of LEDs can indeed be controlled. The brightness of an LED is directly proportional to the current passing through it. Thus, by adjusting the current, we can dim or brighten an LED. This can be achieved by various methods such as pulse-width modulation (PWM), which rapidly turns the LED on and off to control the average amount of light being emitted without changing its color, or by using a current limiting resistor or a constant current source to directly adjust the current. Advanced lighting systems also make use of digital controls to regulate the intensity of LED lighting.

Let's briefly evaluate the other options to understand why they are considered correct:

Option A: LEDs are energy efficient - This is true. LEDs use much less energy compared to traditional incandescent bulbs for the same amount of light output, making them a more energy-efficient lighting solution.

Option B: LEDs have long life if properly manufactured - This statement is correct as well. LEDs can have a very long lifespan, often up to 25,000-50,000 hours, provided they are well-manufactured and used within their specified operating conditions.

Option D: Colours produced by LED do not fade out - LEDs maintain their color throughout their lifespan. Unlike some traditional lighting options that can change color as the filament wears out or the gas in the bulb is depleted, LEDs do not suffer from this issue and thus the colors remain consistent.

Question 112

A thin wire of length 'L' and uniform linear mass density 'm' is bent into a circular coil. The moment of inertia of this coil about tangential axis and in plane of the coil is

Options:

A. $\frac{3\ \mathrm{mL}^2}{5\pi^2}$

B. $\frac{3\ \mathrm{mL}^3}{8\pi^2}$

C. $\frac{3\ \mathrm{mL}^3}{4\pi^2}$

D. $\frac{3\ \mathrm{mL}^2}{7\pi^2}$

Answer: B

Solution:

$\therefore$ Moment of inertia of thin wire:

$$I = \frac{M^2}{2}$$

$M = v \times m$ and $L = 2\pi R$

$$R = \frac{L}{2\pi}$$

$$\therefore \quad I = \frac{Lm}{2}\left(\frac{L}{2\pi}\right)^2$$

$$I = \frac{mL^3}{8\pi^2}$$

$\therefore$ Using Parallel axis theorem:

$$I' = I + MR^2$$

$$I = \frac{mL^3}{8\pi^2} + Lm\left(\frac{L}{2\pi}\right)^2$$

$$I = \frac{3mL^3}{8\pi^2}$$

Question 113

A black body radiates maximum energy at wavelength 'λ' and its emissive power is 'E'. Now due to a change in temperature of that body, it radiates maximum energy at wavelength $\frac{\lambda}{3}$. At that temperature emissive power is

Options:

A. 16 : 1

B. 256 : 1

C. 81 : 1

D. 128 : 1

Answer: C

Solution:

$\therefore$ Emissive power of black body is, From Wien's Law:

$$E = \sigma T^4$$
$$\lambda = \frac{b}{T}$$
$$\therefore \quad E = \frac{\sigma b^4}{\lambda^4}$$
$$\therefore \quad \frac{E_2}{E_1} = \frac{\lambda_1^4}{\lambda_2^4} = \frac{\lambda^4}{\left(\frac{\lambda}{3}\right)^4} = \frac{81}{1}$$

Question 114

If the charge on the capacitor is increased by 3 coulombs, the energy stored in it increases by 44%. The original charge on the capacitor is

Options:

A. 10 C

B. 15 C

C. 20 C

D. 25 C

Answer: B

Solution:

Energy stored in a charged capacitor is

$U = \frac{Q^2}{2C} \quad \Rightarrow U \propto Q^2$

As per given condition, when, $Q_2 = (Q_1 + 3)$,

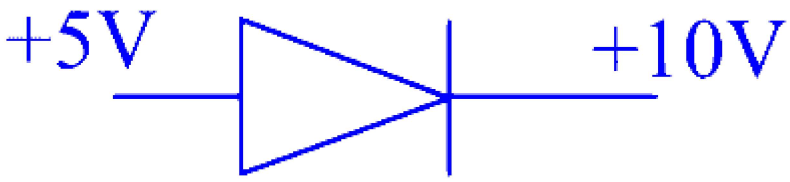
$$\therefore \quad U_2 = 44\% \text{ of } U_1 = \frac{144}{100} U_1$$
$$\therefore \quad \frac{Q_2^2}{2C} = \frac{144}{100} \times \frac{Q_1^2}{2C}$$
$$\therefore \quad Q_1 = \frac{10}{12} Q_2$$
$$\therefore \quad Q_1 = \frac{5}{6}(Q_1 + 3)$$
$$\therefore \quad 6Q_1 = 5Q_1 + 15$$
$$\therefore \quad Q_1 = 15C$$

Question 115

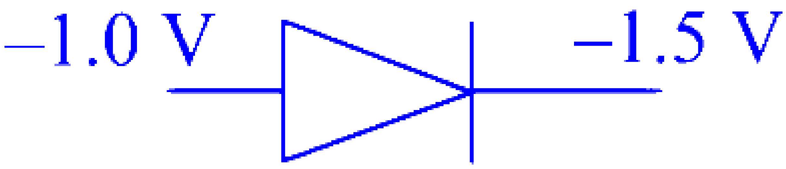
In which figure, the junction diode is forward biased?

Options:

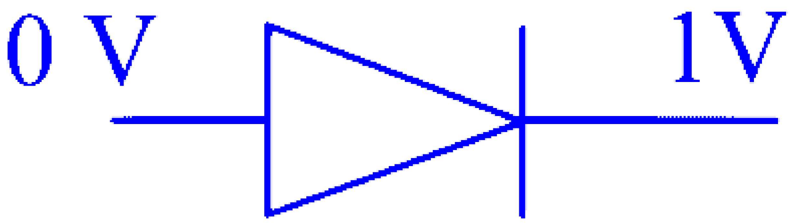
A.



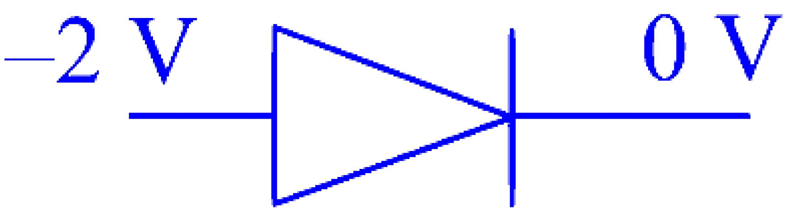
B.



C.

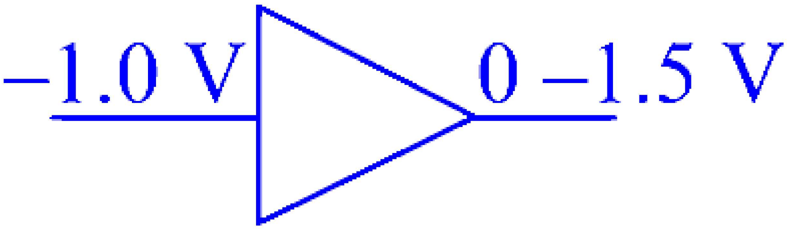


D.



Answer: B

Solution:



Question 116

A particle starts from mean position and performs S.H.M. with period 4 second. At what time its kinetic energy is 50% of total energy?

$$\left(\cos 45^{\circ}=\frac{1}{\sqrt{2}}\right)$$

Options:

A. 0.1 s

B. 0.2 s

C. 0.4 s

D. 0.5 s

Answer: D

Solution:

The total energy is given by $T \cdot E=\frac{1}{2} \mathrm{kA}^2$

Kinetic energy is given by $\mathrm{K} \cdot \mathrm{E}=\frac{1}{2} \mathrm{k}\left(\mathrm{A}^2-\mathrm{x}^2\right)$

$$\begin{array}{ll} & \text{K.E } =\frac{1}{2} P . E \quad \text{...(given)} \\ \therefore & \frac{1}{2} k\left(A^2-x^2\right)=\frac{1}{2}\left(\frac{1}{2} k A^2\right) \\ \therefore & \left(A^2-x^2\right)=\frac{1}{2} A^2 \\ \therefore & \left(A^2-x^2\right)=\frac{1}{2} A^2 \\ \therefore & x=\frac{A}{\sqrt{2}}\end{array}$$

The equation of displacement in SHM is

$$x = A \sin \frac{2\pi t}{T}$$

$$\therefore \quad \frac{A}{\sqrt{2}} = A \sin \frac{2\pi t}{4} \quad \ldots . (\because T = 4 \text{ sec})$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \sin \frac{\pi t}{2}$$

$$\therefore \quad \frac{\pi t}{2} = \sin^{-1} \frac{1}{\sqrt{2}}$$

$$\frac{\pi t}{2} = \frac{\pi}{4}$$

$$\therefore \quad t = 0.5 \text{ s}$$

Question 117

For polyatomic gases, the ratio of molar specific heat at constant pressure to constant volume is (f = degrees of freedom)

Options:

A. $\frac{2+f}{3+f}$

B. $\frac{3+f}{2+f}$

C. $\frac{3+f}{4+f}$

D. $\frac{4+f}{3+f}$

Answer: D

Solution:

For polyatomic gases, molar-specific heat at constant volume is $C_V = (3 + f)R$ and Molar-specific heat at constant pressure is $C_P = (4 + f)R$

$$\therefore \quad \frac{C_P}{C_V} = \frac{4+f}{3+f}$$

Question 118

A particle is moving in a circle with uniform speed 'v'. In moving from a point to another diametrically opposite point

Options:

A. the momentum changes by mv

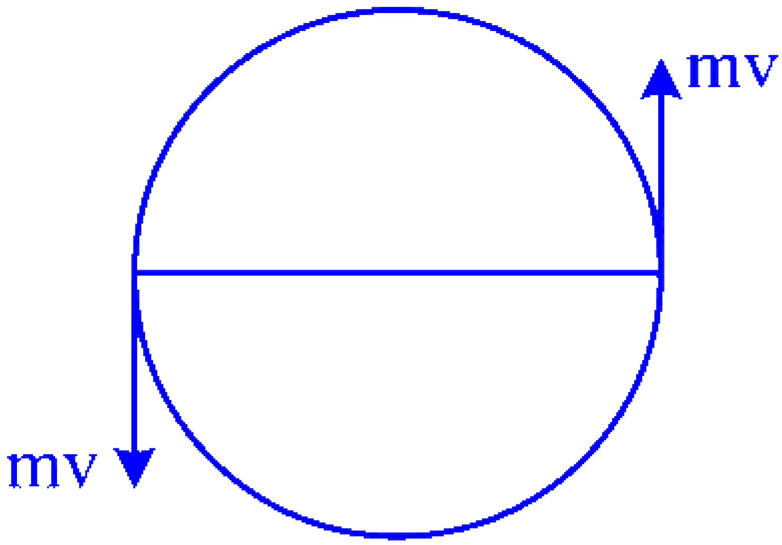
B. the momentum changes by 2 mv

C. the kinetic energy changes by $\frac{1}{2}mv^2$

D. the kinetic energy changes by mv^2

Answer: B

Solution:



At both points, the momentum will be mv, but will be in opposite directions.

$$\therefore \quad \Delta P = mv - (-mv)$$
$$\therefore \quad \Delta P = 2\ mv$$

The momentum changes by 2 mv.

Question 119

Select the WRONG statement from the following. For an isothermal process

Options:

A. Energy exchanged is used to do work

B. Perfect thermal equilibrium with environment

C. Equation of state PV is not constant.

D. No change internal energy.

Answer: C

Solution:

Option C is the incorrect statement for an isothermal process. An isothermal process is one in which the temperature (T) of a system remains constant throughout the process. In thermodynamics, the equation of state for an ideal gas is given by the ideal gas law, which states:

$PV = nRT$

where P is the pressure, V is the volume, n is the number of moles of gas, R is the universal gas constant, and T is the temperature.

In an isothermal process, because the temperature T is constant, the product of P and V must also remain constant for a given amount of gas with a fixed number of moles (n). This is because R is a constant and T is held constant in the process, and thus, the product PV does not change even as pressure and volume do. Therefore,

$PV = constant$

This directly contradicts Option C, which states that the equation of state PV is not constant. Since in an isothermal process for an ideal gas, PV always remains constant if the temperature is constant, Option C is the wrong statement.

Let's verify the other options for an ideal gas undergoing an isothermal process:

Option A is correct because in an isothermal expansion or compression, the energy exchanged with the surroundings is indeed used to do work. No change in internal energy occurs (Option D), due to the fact that internal energy for an ideal gas is a function of temperature alone, and in an isothermal process, temperature does not change. The energy needed to change the volume of the gas comes from or goes into the work done by or on the gas.

Option B is a somewhat ambiguous statement. It can be interpreted as saying the system is in a state of thermal equilibrium with the environment because the temperature remains constant. However, maintaining a perfect thermal equilibrium throughout the process would mean the surroundings are also at the same temperature and capable of supplying or absorbing energy without changing temperature themselves, which is a theoretical idealization.

Nevertheless, Option C is clearly the wrong statement out of the four provided when considering the definition and properties of an isothermal process.

Question 120

Two concentric circular coils of 10 turns each are situated in the same plane. Their radii are $20 \mathrm{~cm}$ and $40 \mathrm{~cm}$ and they carry respectively $0.2 \mathrm{~A}$ and $0.3 \mathrm{~A}$ current in opposite direction. The magnetic field at the centre is ($\mu_0=4 \pi \times 10^{-7}$ SI units)

Options:

A. $4 \pi \times 10^{-7} \mathrm{~T}$

B. $5 \pi \times 10^{-7} \mathrm{~T}$

C. $2 \pi \times 10^{-5} \mathrm{~T}$

D. $7 \pi \times 10^{-6} \mathrm{~T}$

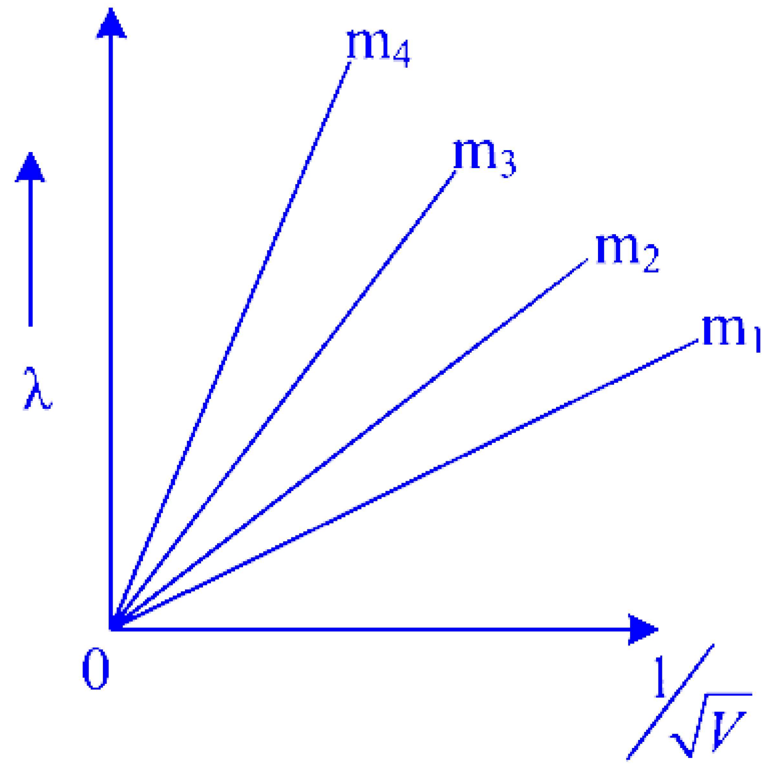
Answer: B

Solution:

$$\begin{aligned} B_{\text{net}} & =\frac{\mu_0 n_1 I_1}{2 r_1}-\frac{\mu_0 n_2 I_2}{2 r_2} \\ & =\frac{10 \mu_0}{2}\left[\frac{0.2}{0.2}-\frac{0.3}{0.4}\right] \\ & =5 \times 4 \pi \times 10^{-7} \times 0.25 \\ & =5 \pi \times 10^{-7} \mathrm{~T}\end{aligned}$$

Question 121

Graph shows the variation of de-Broglie wavelength (λ) versus $\frac{1}{\sqrt{V}}$ where 'V' is the accelerating potential for four particles A, B, C, D carrying same charge but of masses $\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4$. Which on represents a particle of largest mass?



Options:

A. m_1

B. m_2

C. m_3

D. m_4

Answer: A

Solution:

de Broglie wavelength $\lambda = \frac{h}{p}$

$$\therefore \lambda = \frac{h}{\sqrt{2mqv}}$$

$$\therefore \lambda\sqrt{v} = \frac{h}{\sqrt{2mq}} \Rightarrow \frac{\lambda}{\left(\frac{1}{\sqrt{v}}\right)} = \frac{1}{\sqrt{2mq}}$$

$\therefore$ Slope of the graph $= \frac{1}{\sqrt{2mq}}$

The slope will be maximum for minimum mass,

∴ m_4 is minimum and m_1 will be minimum.

Question 122

SI units of self inductance is

Options:

A. $\frac{V-A}{S}$

B. $\frac{V}{A-S}$

C. $\frac{V-S}{A}$

D. $\frac{A}{V-S}$

Answer: C

Solution:

The SI unit of self-inductance is the henry (H). The henry can be defined in terms of other basic SI units (volt, ampere, and second). The correct unit for inductance is based on the definition of inductance in a circuit, which is the ratio of the induced voltage (in volts, V) to the rate of change of current (in amperes per second, A/s). Therefore, the unit of inductance (henry) can be expressed as:

$\frac{V}{(A/s)}$

When we resolve the rate - the division of amperes by seconds - we invert and multiply, which gives us volts times seconds per ampere:

$V \times \frac{s}{A}$

This is equivalent to volt-seconds per ampere, which we write as:

$\frac{V \times s}{A}$

From the options provided, the correct one that matches the unit of self-inductance (henry) is:

Option C: $\frac{V \times s}{A}$

Question 123

When an inductor 'L' and a resistor 'R' in series are connected across a $15 \mathrm{~V}, 50 \mathrm{~Hz}$ a.c. supply, a current of $0.3 \mathrm{~A}$ flows in the circuit. The current differs in phase from applied voltage by $\left(\frac{\pi}{3}\right)^c$. The value of '$R$' is $\left(\sin \frac{\pi}{6}=\cos \frac{\pi}{3}=\frac{1}{2}, \sin \frac{\pi}{3}=\cos \frac{\pi}{6}=\frac{\sqrt{3}}{2}\right)$

Options:

A. 10Ω

B. 15Ω

C. 20Ω

D. 25Ω

Answer: D

Solution:

Given: $E_v=15 \mathrm{~V}, \mathrm{f}=50 \mathrm{~Hz}, \mathrm{I}=0.3 \mathrm{~A}$,

$\phi=\frac{\pi}{3} \mathrm{rad}$

Impedance $Z=\frac{\mathrm{E}_{\mathrm{v}}}{\mathrm{I}}=\frac{15}{0.2}=50 \Omega$

$\tan \phi=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}$

$\tan \frac{\pi}{3}=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}$

$\sqrt{3}=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}$

$\therefore \quad \mathrm{X}_{\mathrm{L}}=\sqrt{3} \mathrm{R}$

Impedance $\mathrm{Z}=\sqrt{\mathrm{R}^2+\mathrm{X}_{\mathrm{L}}^2}$

$\mathrm{Z}=\sqrt{\mathrm{R}^2+(\sqrt{3} \mathrm{R})^2}$

$\mathrm{Z}=\sqrt{4 \mathrm{R}^2}$

$\therefore \quad 2 \mathrm{R}=\mathrm{Z}$

$\therefore \quad \mathrm{R}=\frac{\mathrm{Z}}{2}=\frac{50}{2}=25 \Omega$

Question 124

When an electron is accelerated through a potential 'V', the de-Broglie wavelength associated with it is '4λ'. When the accelerating potential

is increased to 4 V, its wavelength will be

Options:

A. $\frac{\lambda}{4}$

B. $\frac{\lambda}{2}$

C. λ

D. 2λ

Answer: D

Solution:

The de Broglie wavelength (λ) of a particle is given by the formula:

$$\lambda = \frac{h}{\sqrt{2meV}}$$

where:

- h is the Planck's constant,
- m is the mass of the electron,
- e is the charge of the electron, and
- V is the accelerating potential.

When the electron is accelerated through a potential V, the wavelength is given by:

$$\lambda_1 = \frac{h}{\sqrt{2meV}}$$

And the given condition states that this wavelength is 4λ.

When the accelerating potential is increased to $4V$, the new wavelength (λ_2) can be calculated as:

$$\lambda_2 = \frac{h}{\sqrt{2me(4V)}}$$

Let's simplify λ_2:

$$\lambda_2 = \frac{h}{\sqrt{8meV}} = \frac{h}{2\sqrt{2meV}} = \frac{1}{2} \cdot \frac{h}{\sqrt{2meV}}$$

So, $\lambda_2 = \frac{1}{2}\lambda_1 = \frac{1}{2} \cdot 4\lambda = 2\lambda$.

Hence, the answer is Option D: 2λ.

Question 125

Compare the rate of loss of heat from a metal sphere at $627^{\circ}\mathrm{C}$ with the rate of loss of heat from the same sphere at $327^{\circ}\mathrm{C}$, if the temperature of the surrounding is $27^{\circ}\mathrm{C}$. (nearly)

Options:

A. 6.2

B. 5.3

C. 4.8

D. 7.4

Answer: B

Solution:

The heat loss is given as $\mathrm{R}=\mathrm{e}\,\sigma \mathrm{A}\left(\mathrm{T}^4-\mathrm{T}_0^4\right)$

Let surrounding temperature be denoted as T

Heat loss from the metal sphere at a temperature T_1

$\mathrm{R}_1=\mathrm{e} \sigma \mathrm{A}\left(\mathrm{T}_1^4-\mathrm{T}_0^4\right)$

Heat loss from the metal sphere at a temperature T_2

$\mathrm{R}_2=\mathrm{e}\,\sigma \mathrm{A}\left(\mathrm{T}_2^4-\mathrm{T}^4\right)$

$$\therefore \quad \frac{R_1}{R_2}=\frac{T_1^4-T^4}{T_2^4-T^4}=\frac{900^4-300^4}{600^4-300^4}$$

$$\frac{R_1}{R_2}=5.3$$

Question 126

In Balmer series, wavelength of the 2^{nd} line is 'λ_1' and for Paschen series, wavelength of the 1^{st} line is 'λ_2', then the ratio 'λ_1' to 'λ_2' is

Options:

A. $5: 128$

B. 5 : 81

C. 7 : 27

D. 9 : 132

Answer: C

Solution:

For spectral series, $\frac{1}{\lambda}=RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

For the Balmer series, $n_1=2$

The wavelength for 2^{nd} line of the Balmer series is

$$\frac{1}{\lambda_1}=RZ^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$
$$\frac{1}{\lambda_1}=RZ^2\left(\frac{1}{4}-\frac{1}{16}\right)$$
$$\frac{1}{\lambda_1}=RZ^2\left(\frac{3}{16}\right)\Rightarrow\lambda_1=\left[\frac{16}{3}\right]$$

For the Paschen series, $n_1=3$

The wavelength for 1^{st} line of the Paschen series is

$$\frac{1}{\lambda_2}=RZ^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right)$$
$$\frac{1}{\lambda_2}=RZ^2\left(\frac{1}{9}-\frac{1}{16}\right)$$
$$\lambda_2=\frac{144}{7}$$
$$\frac{1}{\lambda_2}=RZ^2\left(\frac{7}{144}\right)$$
$$\therefore\quad\frac{\lambda_1}{\lambda_2}=\frac{16}{3}\times\frac{7}{144}=\frac{7}{27}$$

Question 127

A coil of 'n' turns and radius 'R' carries a current 'I'. It is unwound and rewound again to make another coil of radius $\left(\frac{R}{3}\right)$, current

remaining the same. The ratio of magnetic moment of the new coil to that of original coil is

Options:

A. 3 : 1

B. 1 : 3

C. 9 : 1

D. 1 : 9

Answer: B

Solution:

As the length of the wire remains the same, we can write

$$N_1 2\pi R = N_2 \times 2\pi \frac{R}{3}$$
$$N_1 = \frac{N_2}{3}$$
$$N_2 = 3N_1$$

Magnetic moment of coil $\mu =$ NIA

$\therefore \quad \mu_1 = N_1 I A_1 = N_1 I \pi R^2$

$\therefore \quad \mu_2 = N_2 I A_2 = 3\frac{N_1 I \pi R^2}{9}$

$\therefore \quad \frac{\mu_2}{\mu_1} = 3\frac{N_1 I \pi R^2}{9} \times \frac{1}{N_1 I \pi R^2} = \frac{1}{3}$

Question 128

An ink mark is made on a piece of paper on which a glass slab of thickness 't' is placed. The ink mark appears to be raised up through a distance 'x' when viewed at nearly normal incidence. If the refractive index of the material of glass slab is 'μ' then the thickness of glass slab is given by

Options:

A. $\frac{\mu x}{\mu - 1}$

B. $\frac{(\mu-1)}{\mu}$

C. $\frac{\mu-1}{\mu \mathrm{x}}$

D. $\frac{\mu}{(\mu-1) x}$

Answer: A

Solution:

Here, the normal shift is x

The formula for the normal shift is

$$
\begin{aligned}
& x=t\left(1-\frac{1}{\mu}\right) \\
& t=\frac{x}{\left(1-\frac{1}{\mu}\right)} \\
& x=t\left(1-\frac{1}{\mu}\right) \\
& t=\frac{x}{\left(1-\frac{1}{\mu}\right)} \\
\therefore \quad & t=\frac{x \mu}{(\mu-1)}
\end{aligned}
$$

Question 129

A spherical metal ball of radius 'r' falls through viscous liquid with velocity 'V'. Another metal ball of same material but of radius $\left(\frac{r}{3}\right)$ falls through same liquid, then its terminal velocity will be

Options:

A. $\frac{\mathrm{V}}{3}$

B. $\frac{V}{4}$

C. $\frac{\mathrm{V}}{6}$

D. $\frac{V}{9}$

Answer: D

Solution:

$v=\frac{2r^2(\rho-\sigma)g}{9\Delta}$

ρ, σ and η are constant .... (given)

$\Rightarrow \mathrm{v} \propto \mathrm{r}^2$

The ratio of terminal velocities is

$$\frac{v_1}{v_2}=\frac{r_1^2}{r_2^2}$$
$$\frac{v_1}{v_2}=\frac{r^2}{\left(\frac{r}{3}\right)^2}$$
$$\frac{v_1}{v_2}=9$$
$$\therefore \quad v_2=\frac{v}{9}$$

Question 130

A body of mass 'm' attached at the end of a string is just completing the loop in a vertical circle. The apparent weight of the body at the lowest point in its path is (g = gravitational acceleration)

Options:

A. zero

B. mg

C. 3 mg

D. 6 mg

Answer: D

Solution:

The tension at the lowest point is $\mathrm{T}=\frac{\mathrm{mv}^2}{\mathrm{r}}+\mathrm{mg}$

To complete the vertical circle, the minimum velocity should be $v = \sqrt{5gr}$

$$\therefore \quad T = \frac{m(\sqrt{5gr})^2}{r} + mg$$
$$R = 5mg + mg$$
$$\therefore \quad T = 6mg$$

Question 131

Select the WRONG statement from the following. In a streamline flow

Options:

A. velocity of a fluid at a given point is never constant.

B. velocity is smaller than critical velocity.

C. layers are always parallel.

D. the particles do not move in random direction.

Answer: A

Solution:

In streamlined flow, the velocity of a fluid at a specific point is always constant because each particle of the fluid follows a well-defined path and there is no mixing or crossing of paths between particles.

Question 132

Two point charges '$q1$' and '$q2$' are separated by a distance 'd'. What is the increase in potential energy of the system when '$q2$' is moved towards '$q1$' by a distance 'x' ? $(x < d)(\frac{1}{4\pi\varepsilon_0} = K$, constant)

Options:

A. $-\frac{Kq_1q_2x}{d(d-x)}$

B. $-\frac{Kq_1q_2}{d(d-x)}$

C. $\frac{\mathrm{Kq_1q_{2x}}}{\left(\mathrm{d}^2-\mathrm{x}^2\right)}$

D. $\frac{\mathrm{Kq_1q_2x}}{(\mathrm{d}^2-\mathrm{x}^2)}$

Answer: A

Solution:

The potential energy between two charges is given as $U=\frac{kq_1q_2}{r}$

Initial potential energy is $U_\mathrm{f}=\frac{kq_1q_2}{r}$

When charge q_2 moves towards the q_1 the separation between the charges becomes $\mathrm{d}-\mathrm{x}$

The final potential energy is $U_\mathrm{f}=\frac{\mathrm{kq_1q_2}}{(\mathrm{~d}-\mathrm{x})}$

The increase in potential energy is

$$\therefore \quad \Delta \mathrm{U}=\mathrm{U_f}-\mathrm{U_f}$$
$$\therefore \quad \Delta U=\frac{kq_1q_2}{d}-\frac{k_1q_2}{(d-x)}$$
$$\therefore \quad \Delta U=\mathrm{kq_1q_2}\left(\frac{1}{\mathrm{d}}-\frac{1}{\mathrm{d}-\mathrm{x}}\right)$$
$$\therefore \quad \Delta U=\frac{-\mathrm{kq_1q_2x}}{\mathrm{d}(\mathrm{d}-\mathrm{x})}$$

Question 133

The ratio of intensities of two points on a screen in Young's double slit experiment when waves from the two slits have a path difference of $\frac{\lambda}{4}$ and $\frac{\lambda}{6}$ is

$$\left(\cos 90^{\circ}=0, \cos 60^{\circ}=0.5\right)$$

Options:

A. 2 : 1

B. 2 : 3

C. 3 : 4

D. 3 : 5

Answer: B

Solution:

The intensity at the point due to interference is given as $I=I_1+I_2+2\sqrt{I_1 I_2}\cos\phi$ ..... (i)

For path difference $\frac{\lambda}{4}$, the phase difference is

$\phi_1=\frac{2\pi}{\lambda}\times\frac{\lambda}{4}=\frac{\pi}{2}$

For path difference $\frac{\lambda}{6}$, the phase difference is

$\phi_2=\frac{2\pi}{\lambda}\times\frac{\lambda}{6}=\frac{\pi}{3}$

Assuming equal intensity of the interfering waves i.e., $I_1=I_2=I_0$

Equation (i) becomes,

$$\mathrm{I}=\mathrm{I}_0+\mathrm{I}_0+2\mathrm{I}_0\cos\phi$$
$$\mathrm{I}=2\mathrm{I}_0(1+\cos\phi)$$

For the given path difference, $I_1=2I_0\left(1+\cos\frac{\pi}{2}\right)$, and $I_2=2I_0\left(1+\cos\frac{\pi}{3}\right)$

$\therefore \quad \frac{I_1}{I_2}=\frac{1+\cos\frac{\pi}{2}}{1+\cos\frac{\pi}{3}}$

$$\frac{\mathrm{I}_1}{\mathrm{I}_2}=\frac{1+0}{1+0.5}$$
$$\therefore \quad \frac{\mathrm{I}_1}{\mathrm{I}_2}=\frac{1}{1.5}=\frac{2}{3}$$

Question 134

A simple pendulum is oscillating with frequency 'F' on the surface of the earth. It is taken to a depth $\frac{\mathrm{R}}{3}$ below the surface of earth. ($\mathrm{R}=$ radius of earth). The frequency of oscillation at depth $\mathrm{R}/3$ is

Options:

A. $\frac{2F}{3}$

B. $\frac{F}{\sqrt{1.5}}$

C. F

D. $\frac{F}{3}$

Answer: B

Solution:

The frequency of the pendulum at the surface is given as

$f=\frac{1}{2\pi}\sqrt{\frac{g}{l}}$

At depth the formula for gravitational acceleration is $g_{\text{eff}}=g\left(1-\frac{d}{R}\right)$

For $d=\frac{R}{3}$, $g_{\text{eff}}=g\left(1-\frac{1}{3}\right)$

The frequency at depth $d=\frac{R}{3}$

$f_d=\frac{1}{2\pi}\sqrt{\frac{g\left(1-\frac{1}{3}\right)}{l}}=\frac{1}{2\pi}\sqrt{\frac{2g}{3l}}$

Take the ratio of both frequencies

$$\frac{f_d}{f}=\sqrt{\frac{2}{3}}$$
$$\therefore\quad f_d=\frac{F}{\sqrt{1.5}}\quad\ldots.(\because f=F)$$

Question 135

An a.c. source of $15\text{ V}, 50\text{ Hz}$ is connected across an inductor (L) and resistance (R) in series R.M.S. current of 0.5 A flows in the circuit. The phase difference between applied voltage and current is $\left(\frac{\pi}{3}\right)$ radian. The value of resistance (R) is $\left(\tan 60^{\circ}=\sqrt{3}\right)$

Options:

A. 10Ω

B. 12Ω

C. 15Ω

D. 20Ω

Answer: C

Solution:

Given data: $\mathrm{E}=15 \mathrm{~V}, \mathrm{f}=50 \mathrm{~Hz}, \mathrm{I}=0.5 \mathrm{~A}, \phi=\frac{\pi}{3}$ rad

Impedance is given as $Z=\frac{E}{I}=\frac{15}{0.5}=30\Omega$

$$\tan \phi=\frac{\mathrm{X_L}}{\mathrm{R}}$$
$$\tan \frac{\pi}{3}=\frac{\mathrm{X_L}}{\mathrm{R}}$$
$$\sqrt{3}=\frac{\mathrm{X_L}}{\mathrm{R}}$$
$$\therefore \quad \mathrm{X_L}=\sqrt{3}\mathrm{R}$$

The formula for impedance is

$$Z=\sqrt{R^2+X_L^2}$$
$$Z=\sqrt{R^2+(\sqrt{3}R)^2}$$
$$Z=\sqrt{4R^2}$$
$$\therefore \quad 2R=Z$$
$$\therefore \quad R=\frac{Z}{2}=\frac{30}{2}=15\Omega$$

Question 136

A ball is projected vertically upwards from ground. It reaches a height 'h' in time t_1, continues its motion and then takes a time t_2 to reach ground. The height h in terms of g, t_1 and t_2 is ($\mathrm{g}=$ acceleration due to gravity)

Options:

A. $\frac{1}{2}\frac{gt_1}{t_2}$

B. $\frac{1}{2}gt_1t_2$

C. gt_1t_2

D. $2\ gt_1t_2$

Answer: B

Solution:

We know,

$S = ut + \frac{1}{2}at^2$

The total time required for the ball to go up and reach the ground is $t = t_1 + t_2$, and the total displacement is zero.

$$\therefore \quad 0 = u(t_1 + t_2) + \frac{1}{2}g(t_1 + t_2)^2$$
$$\therefore \quad u = \frac{1}{2}g(t_1 + t_2)$$

The displacement in time t_1 is

$$h = \frac{1}{2}g(t_1 + t_2)t_1 - \frac{1}{2}gt_1^2$$
$$h = \frac{1}{2}gt_1(t_1 + t_2 - t_1)$$
$$\therefore \quad h = \frac{1}{2}gt_1t_2$$

Question 137

The volume of a metal block increases by 0.225% when its temperature is increased by $30^\circ C$. Hence coefficient of linear expansion of the material of metal block is

Options:

A. $7.5 \times 10^{-5}/^\circ C.$

B. $6.75 \times 10^{-5}/^\circ C.$

C. $2.5 \times 10^{-5} / {}^{\circ}\mathrm{C}$.

D. $1.5 \times 10^{-5} / {}^{\circ}\mathrm{C}$.

Answer: C

Solution:

The increase in volume is $\frac{\Delta \mathrm{V}}{\mathrm{V}} \times 100 = 0.225$

Change in temperature is $\Delta \mathrm{T} = 30^{\circ}\mathrm{C}$

We know, $\gamma = 3\alpha$

$\therefore$ The change in volume is $\Delta \mathrm{V} = \mathrm{V}\gamma\Delta \mathrm{T}$

$$\therefore \quad \frac{\Delta V}{V} \times 100 = 3\alpha\Delta \mathrm{T} \times 100$$
$$\therefore \quad 0.225 = 3\alpha \times 30 \times 100$$
$$\therefore \quad \alpha = \frac{0.225}{3 \times 30 \times 100}$$
$$\therefore \quad \alpha = 2.5 \times 10^{-5} / {}^{\circ}\mathrm{C}$$

Question 138

A tuning fork gives 3 beats with $50 \mathrm{~cm}$ length of sonometer wire. If the length of the wire is shortened by $1 \mathrm{~cm}$, the number of beats is still the same. The frequency of the fork is

Options:

A. 256 Hz

B. 288 Hz

C. 297 Hz

D. 320 Hz

Answer: C

Solution:

The frequency of a vibrating wire $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$\therefore \quad \mathrm{f} \propto \frac{1}{l} \Rightarrow \mathrm{fl} = \text{constant}$

Let the frequency of the fork be f and the initial and final frequencies of the wire be f_1 and f_2.

The number of beats heard before decreasing the length is $f - f_1 = 3$ .... (i)

The number of beats after decreasing the length is $\mathrm{f_2} - \mathrm{f} = 3$ .... (ii)

$\therefore \quad \mathrm{f_1} l_1 = \mathrm{f_2} l_2$

$\therefore (\mathrm{f} - 3)50 = (\mathrm{f} + 3)49$ ... from (i) and (ii)

$50\mathrm{f} - 49\mathrm{f} = 147 + 150$

$\therefore \quad \mathrm{f} = 297\ \mathrm{Hz}$

Question 139

The depth at which acceleration due to gravity becomes $\frac{\mathrm{g}}{2\mathrm{n}}$ is $(\mathrm{R} =$ radius of earth, $\mathrm{g} =$ acceleration due to gravity on earth's surface, n is integer)

Options:

A. $\frac{\mathrm{R}(1-2\mathrm{n})}{\mathrm{n}}$

B. $\frac{\mathrm{R}(1-\mathrm{n})}{2\mathrm{n}}$

C. $\frac{\mathrm{R}(\mathrm{n}-1)}{\mathrm{n}}$

D. $\frac{\mathrm{R}(2\mathrm{n}-1)}{2\mathrm{n}}$

Answer: D

Solution:

The gravitational acceleration at depth is given as $g_d = g\left[1 - \frac{d}{R}\right]$

Given $\mathrm{g_d} = \frac{\mathrm{g}}{2\mathrm{n}}$

$$\therefore \quad \frac{g}{2n} = g\left[1 - \frac{d}{R}\right]$$

$$\frac{d}{R} = 1 - \frac{1}{2n}$$

$$d = \left[\frac{2n-1}{2n}\right]R$$

Question 140

Two resistance X and Y are connected in the two gaps of a meterbridge and the null points is obtained at $20\ \mathrm{cm}$ from zero end. When the resistance of 20Ω is connected in series with the smaller of the two resistance X and Y, the null point shifts to $40\ \mathrm{cm}$ from left end. The value of smaller resistance in ohm is

Options:

A. 6

B. 9

C. 12

D. 15

Answer: C

Solution:

For a meterbridge, $\frac{\mathrm{X}}{\mathrm{Y}} = \frac{l}{100-l}$

In the first case, $l = 20$ cm

$$\frac{X}{Y} = \frac{20}{100-20} = \frac{20}{80} = \frac{1}{4}$$

$$\therefore \quad 4X = Y$$

In the second case, $l' = 40$ cm

$\therefore \quad \frac{X'}{Y'} = \frac{40}{100-40} = \frac{40}{60} = \frac{2}{3}$

But, $X' = X + 20$ and $Y' = Y$

$\therefore \quad \frac{X+20}{Y} = \frac{2}{3}$

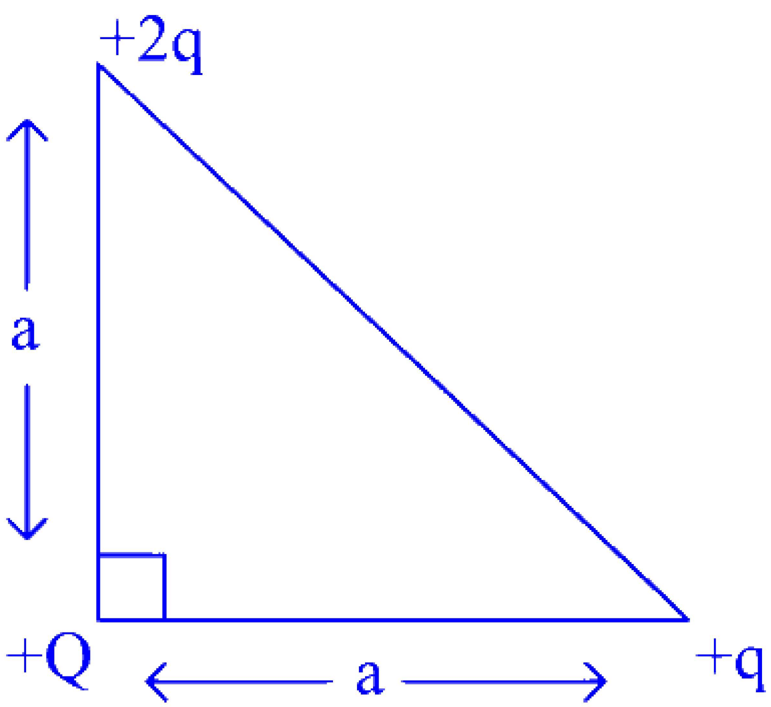
$\therefore \quad \frac{X+20}{4X} = \frac{2}{3}$

$\therefore \quad 8X = 3X + 60$

$\therefore \quad X = 12\Omega$

Question 141

Three point charges +Q, +2q and +q are placed at the vertices of a right angled isosceles triangle. The net electrostatic potential energy of the configuration is zero, if Q is equal to



Options:

A. $-\frac{\sqrt{2}}{3}q$

B. $+\frac{\sqrt{2}}{3}q$

C. $-\frac{3}{\sqrt{2}}q$

D. $+\frac{3}{\sqrt{2}}$q

Answer: A

Solution:

Net electrostatic potential energy of the system is,

$$U=\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{a}+\frac{2Qq}{a}+\frac{2qq}{\sqrt{2}a}\right)=0$$
$$Q+2Q+\frac{2q}{\sqrt{2}}=0$$
$$3Q+\sqrt{2}q=0$$
$$\therefore\quad Q=\frac{-\sqrt{2}q}{3}$$

Question 142

Resistor of 2Ω, inductor of 100μH and capacitor of 400pF are connected in series across a source of $\mathrm{e_{rms}}=0.1$ Volt. At resonance, voltage drop across inductor is

Options:

A. 25 V

B. 2.5 V

C. 250 V

D. 20 V

Answer: A

Solution:

At resonance condition, $\mathrm{X_C}=\mathrm{X_L}$

The impedance is given as:

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$
$$\therefore \quad Z=R=2\Omega$$
$$I_{rms}=\frac{e_{ms}}{R}$$
$$I_{rms}=\frac{0.1}{2}=0.05\text{ A}$$
$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{10^{-4}\times 4\times 10^{-10}}}$$
$$\omega=5\times 10^{6}$$

$\therefore$ The voltage-drop across the inductor is,

$$V=I_{\text{rms}}\times X_L=I_{\text{rms}}\times L\omega$$
$$V=0.05\times 10^{-4}\times 5\times 10^{6}$$
$$V=25\text{ V}$$

Question 143

An electron makes a full rotation in a circle of radius 0.8 m in one second. The magnetic field at the centre of the circle is $(\mu_0=4\pi\times 10^{-7}$ SI units)

Options:

A. $4\pi\times 10^{-26}$ T

B. $2\pi\times 10^{-26}$ T

C. $4\pi\times 10^{-19}$ T

D. $2\pi\times 10^{-19}$ T

Answer: A

Solution:

$$\omega=\frac{2\pi}{T}$$
$$I=\frac{q\omega}{2\pi}=\frac{1.6\times 10^{-19}\times 2\pi}{2\pi}\quad \ldots\left(\because I=\frac{q}{t}\right)$$
$$I=1.6\times 10^{-19}\text{ A}$$

$\therefore$ The magnetic field at the centre of the circle is:

$$B = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19}}{2 \times 0.8}$$
$$B = 4\pi \times 10^{-26}\ T$$

Question 144

In Young's double slit experiment when a glass plate of refractive index 1.44 is introduced in the path of one of the interfering beams, the fringes are displaced by a distance 'y'. If this plate is replaced by another plate of same thickness but of refractive index 1.66, the fringes will be displaced by a distance

Options:

A. $\frac{3y}{2}$

B. $\frac{2y}{3}$

C. $\frac{5y}{4}$

D. $\frac{4y}{5}$

Answer: A

Solution:

As a glass plate is used in one of the paths,

$$y_1 = \frac{\beta}{\lambda}(1.44 - 1)t$$
$$y_1 = 0.44t \times \frac{\beta}{\lambda}$$

New displacement is:

$$y_2 = \frac{\beta}{\lambda}(1.66 - 1)t$$
$$y_2 = 0.66t \times \frac{\beta}{\lambda}$$
$$\frac{y_2}{y_1} = \frac{0.66}{0.44}$$
$$\therefore \quad y_2 = \frac{3y}{2}$$

Question 145

A monoatomic ideal gas initially at temperature '$\mathrm{T_1}$' is enclosed in a cylinder fitted with massless, frictionless piston. By releasing the piston suddenly the gas is allowed to expand to adiabatically to a temperature '$\mathrm{T_2}$'. If '$\mathrm{L_1}$' and '$\mathrm{L_2}$' are the lengths of the gas columns before and after expansion respectively, then $\frac{\mathrm{T_2}}{\mathrm{T_1}}$ is

Options:

A. $\frac{\mathrm{L_1}}{\mathrm{L_2}}$

B. $\frac{\mathrm{L_2}}{\mathrm{L_1}}$

C. $\left(\frac{\mathrm{L_1}}{\mathrm{L_2}}\right)^{2/3}$

D. $\left(\frac{\mathrm{L_2}}{\mathrm{L_1}}\right)^{2/3}$

Answer: C

Solution:

For an adiabatic process

$$\mathrm{T_1\,V_1^{\gamma-1} = T_2\,V_2^{\gamma-1}}$$

$$\therefore \quad \frac{\mathrm{T_2}}{\mathrm{T_1}} = \left(\frac{\mathrm{V_1}}{\mathrm{V_2}}\right)^{\gamma-1}$$

For a monoatomic gas, $r = \frac{5}{3}$

$$\Rightarrow \gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3}$$

$$V_1 = AL_1 \text{ and } V_2 = AL_2$$

$$\therefore \quad \frac{T_2}{T_1} = \left[\frac{AL_1}{AL_2}\right]^{2/3} = \left[\frac{L_1}{L_2}\right]^{2/3}$$

$$\gamma - 1 = \frac{2}{3}$$

For an adiabatic process,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_1}{V_2}\right)^{\frac{2}{3}}$$
$$V \propto L$$
$$\frac{T_2}{T_1} = \left(\frac{L_1}{L_2}\right)^{\frac{2}{3}}$$

Question 146

A tuning fork of frequency 220 Hz produces sound waves of wavelength 1.5 m in air at N.T.P. The increase in wavelength when the temperature of air is 27°C is nearly $\left(\sqrt{\frac{300}{273}} = 1.05\right)$

Options:

A. 0.06 m

B. 0.10 m

C. 0.09 m

D. 0.07 m

Answer: D

Solution:

$$v_0 = f\lambda_0 = 220 \times 1.5$$
$$v_0 = 330 \text{ m/s}$$

We know,

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}}$$

$$\Rightarrow v = 330\sqrt{\frac{300}{273}} = 330 \times 1.05$$
$$v = 346.1 \text{ m/s}$$
$$\therefore \quad \lambda = \frac{v}{f} = \frac{346.1}{220}$$
$$\lambda = 1.57 \text{ m}$$

$\therefore$ The increase in wavelength is:

$$\Delta\lambda = \lambda - \lambda_0 = 1.57 - 1.5$$
$$\Delta\lambda = 0.07 \text{ m}$$

Question 147

An air craft of wing span 40 m files horizontally in earth's magnetic field $5 \times 10^{-5} \text{ T}$ at a speed of 500 m/s. The e.m.f. generated between the tips of the wings of the air craft is

Options:

A. 0.5 V

B. 1 V

C. 1.2 V

D. 1.5 V

Answer: B

Solution:

The emf generated between the tips of the wings of the air craft is,

$$\varepsilon = \text{Blv}$$
$$\varepsilon = 5 \times 10^{-5} \times 40 \times 500$$
$$\varepsilon = 1 \text{ V}$$

Question 148

In P^{th} second, a particle describes angular displacement of 'β' rad. If it starts from rest, the angular acceleration is

Options:

A. $\frac{\beta}{P}$

B. $\frac{\beta}{(\mathrm{P}-1)}$

C. $\frac{2\beta}{(2\mathrm{P}-1)}$

D. $\frac{(2\beta+1)}{(2\mathrm{P}-1)}$

Answer: C

Solution:

The equation for the angular displacement of the particle that starts from rest is given as:

$\beta = 0 + \frac{1}{2}\alpha(2\mathrm{P} - 1)$

$\therefore$ The angular acceleration of the particle is $\alpha = \frac{2\beta}{(2\mathrm{P}-1)}$

Question 149

Inductance per unit length near the middle of a long solenoid is ($\mu_0 =$ permeability of free space, $\mathrm{n} =$ number of turns per unit length, $\mathrm{d} =$ the diameter of the solenoid)

Options:

A. $\mu_0\pi\left(\frac{\mathrm{nd}}{2}\right)^2$

B. $4\mu_0\pi\left(\frac{\mathrm{nd}}{2}\right)$

C. $\left(\frac{\mu_0\pi nd}{2}\right)$

D. $\frac{4\mu_0\pi}{\mathrm{n}^2\ \mathrm{d}^2}$

Answer: A

Solution:

The inductance long solenoid is

$$L=\frac{\mu_0 N^2 A}{l}$$

$$L=\mu_0\left(\frac{N}{l}\right)^2\times\pi\times\frac{d^2}{4}$$

$\therefore$ The inductance per unit length near the middle of a long solenoid is:

$\frac{L}{l}=\mu_0\pi\left(\frac{nd}{2}\right)^2 \quad \ldots.(\because \frac{N}{l}=n)$

Question 150

One of the slits in Young's double slit experiment is covered with a transparent sheet of thickness 2.9×10^{-3} cm. The central fringe shifts to a position originally occupied by the 25^{th} bright fringe. If $\lambda=5800$ $\overset{o}{A}$, the refractive index of the sheet is

Options:

A. 1.65

B. 1.60

C. 1.55

D. 1.50

Answer: D

Solution:

As it is given that a transparent sheet of certain thickness is inserted, we use

$$(\mu-1)\times t=N\lambda$$
$$25\lambda=(\mu-1)t$$
$$\mu-1=\frac{25\lambda}{t}$$

$\therefore$ The refractive index of the sheet is:

$$\mu=\frac{25\times5800\times10^{-10}}{2.9\times10^{-5}}+1$$
$$\mu=1.50$$